

2019

ESE

Engineering Service Examination

# Mechanical Engineering

A photograph of an industrial facility, likely a refinery or chemical plant, at night. The scene is illuminated by numerous lights, creating a complex network of structures, towers, and pipes against a dark blue sky. The foreground shows a fence and some lower-level buildings.

## Industrial Engineering

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# **ESE**

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# **2019**

**MACHINE DESIGN**

**MECHANICAL ENGINEERING**



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Publications



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**ESE-2019:** Machine Design| Detailed theory with GATE & ESE previous year papers and detailed solutions.

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**CHAPTER - 1*****STATIC AND DYNAMIC LOADING*****1.1 INTRODUCTION**

Machine design is defined as the use of scientific principles, technical information and imagination in the dissipation of a machine

Or a mechanical system to perform specific functions with maximum economy and efficiency.

**1.2 BASIC PROCEDURE OF MACHINE DESIGN**

**Example.** Gear box assembly

**1.3 BASIC REQUIREMENTS OF MACHINE ELEMENTS****1. Strength**

**2. Rigidity:** A machine component should be rigid and it should not deflect or bend too much due to forces or moments that acts on it. For example, a transmission shaft is many times designed on the basis of lateral and torsional rigidities. Therefore, maximum permissible deflection and maximum permissible angle of twist are the criterion of Design.

**3. Wear Resistance:** Wear is the main reason for putting the machine part out of order. It reduces useful life of the component. Wear also leads to loss accuracy of machine tools. Surface hardening is generally applied to increase wear resistance.

**4. Minimum Dimensions & Weight:** Material should be strong, hard and rigid with minimum possible dimensions and weight. This will result in minimum material cost.

**5. Manufacturability:** It is the ease of fabrication and assembly so that labour cost may be minimized.

**6. Safety:** The shape and dimensions of the machine parts should ensure safety to the operator of the machine.

**7. Conformance to Standards:** It should conform to national and international standards covering its possible dimensions, grade and material.

**8. Reliability:** It is the probability that machine part will perform its intended functions under desired operating conditions over specified period of time.

**9. Maintainability:** It is case by which a machine part can be serviced or repaired.

# GATE QUESTIONS

1. Pre-tensioning of a bolted joint is used to  
 (a) Strain harden the bolt head  
 (b) Decrease stiffness of the bolted joint  
 (c) Increase stiffness of the bolted joint  
 (d) Prevent yielding of the thread root

[GATE - 2018]

2. Fatigue life of a material for a fully reversed loading condition is estimated from  $\sigma_a = 1100 N^{-0.15}$  where  $\sigma_a$  is the stress amplitude in MPa and N is the failure life in cycles. The maximum allowable stress amplitude (in MPa) for a life of  $1 \times 10^5$  cycles under the same loading condition is \_\_\_\_\_ (correction to two decimal places).

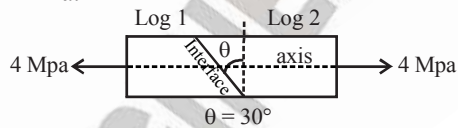
[GATE - 2018]

3. If  $\sigma_1$  and  $\sigma_3$  are maximum and minimum values of principle stresses algebraically then the maximum value of shear stress is ?

[GATE - 2018]

- (a)  $\frac{\sigma_1 - \sigma_3}{2}$                       (b)  $\sqrt{\frac{\sigma_1 - \sigma_3}{2}}$   
 (c)  $\left(\frac{\sigma_1 + \sigma_3}{2}\right)$                       (d)  $\sqrt{\frac{\sigma_1 + \sigma_3}{2}}$

4. Two wooden pieces are attached as shown in figure below. Their attached with figure so the angle ( $\theta$ ) is given in the diagram is  $30^\circ$  and the whole assembly experience 10 in tensile stress of 4 MPa.



1. Maximum tensile stress glue can take 2.5 Mpa  
 2. Shear stress glue can take 1.5 Mpa  
 Assume that failure will be happen in glue not in wood ?

[GATE - 2018]

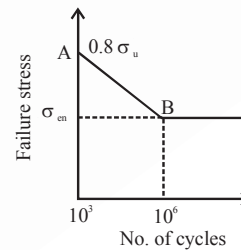
- (a) It fails by two tensile stress not shear stress  
 (b) It fails by shear stress not tensile ?

- (c) Fail by both of them  
 (d) Fail by none of them

5. A bar is subjected to a combination of a steady load of 60 kN and a load fluctuating between - 10 kN and 90 kN. The corrected endurance limit of the bar is 150 MPa. The yielded strength of the material is 480 MPa and he ultimate strength of the material is 600 MPa. The bar cross-section is square with side a. if the factor of safety is 2, the value if a (in mm), according to the modified Goodman's criterion, is \_\_\_\_\_ (correct to two decimal places).

[GATE - 2017]

6. A machine element has an ultimate strength ( $\sigma_u$ ) of  $600 \text{ N/mm}^2$ , and endurance limit ( $\sigma_{en}$ ) of  $250 \text{ N/mm}^2$ . The fatigue curve for the element on a log - log plot is shown below. If the element is to be designed for a finite life of 10000 cycles, the maximum amplitude of a completely reversed operating stress is \_\_\_\_\_  $\text{N/mm}^2$



[GATE - 2017]

7. The principal stresses at a point in a critical section of machine component are  $\sigma_1 = 60 \text{ MPa}$ ,  $\sigma_2 = 5 \text{ MPa}$  and  $\sigma_3 = -40 \text{ MPa}$ . For the material of the component, the tensile yield strength is  $\sigma_y = 200 \text{ MPa}$ . According to the maximum shear theory, the factory of safety is

[GATE - 2017]

- (a) 1.67                                      (b) 2.00  
 (c) 3.60                                      (d) 4.00

## ESE OBJ QUESTIONS

1. A machine component is subjected to a flexural stress, which fluctuates between  $300 \text{ MN/m}^2$  and  $-150 \text{ MN/m}^2$ . Taking the yield strength = 0.55 of the ultimate strength, endurance strength = 0.50 of the ultimate strength and factor of safety to be 2, the value of the minimum ultimate strength according to modified Goodman relation will be
- [ESE - 2017]
- (a)  $1100 \text{ MN/m}^2$                       (b)  $1075 \text{ MN/m}^2$   
(c)  $1050 \text{ MN/m}^2$                       (d)  $1025 \text{ MN/m}^2$
2. Consider the following statements:  
For a component made of ductile material, the failure criterion will be
1. Endurance limit, if the external force is fluctuating
  2. Fatigue, if the external force is fluctuating
  3. Yield stress, if the external force is static
- Which of the above statements are correct
- [ESE - 2017]
- (a) 1 and 2 only                      (b) 1 and 3 only  
(c) 2 and 3 only                      (d) 1, 2 and 3
3. Consider the following statements:  
On heating an elastomer under tensile load, its shrinkage
1. maximizes the enthalpy
  2. maximizes the entropy
  3. minimizes the free energy
  4. avoids breaking
- Which of the above statements are correct?
- [ESE - 2017]
- (a) 1 and 2                      (b) 2 and 3  
(c) 3 and 4                      (d) 1 and 4
4. **Statement(I)** : Directionally solidified materials have good creep resistance.  
**Statement (II)**: Directionally solidified materials may be so loaded that there is no shearing stress along, or tensile stress across, the grain boundaries.
- [ESE - 2017]
- (a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)  
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)  
(c) Statement (I) is true but Statement (II) is false.  
(d) Statement (I) is false but Statement (II) is true.
5. A solid shaft is designed to transmit  $100 \text{ kW}$  while rotating at  $N \text{ r.p.m.}$  If the diameter of the shaft is doubled and is allowed to operate at  $2N \text{ r.p.m.}$ , the power that can be transmitted by the latter shaft is
- [ESE - 2016]
- (a)  $200 \text{ kW}$                       (b)  $400 \text{ kW}$   
(c)  $800 \text{ kW}$                       (d)  $1600 \text{ kW}$
6. The diameter of a shaft to transmit  $25 \text{ kW}$  at  $1500 \text{ r.p.m.}$  given that the ultimate strength is  $150 \text{ MPa}$  and the factor of safety is 3, will nearly be
- [ESE - 2016]
- (a)  $12 \text{ mm}$                       (b)  $16 \text{ mm}$   
(c)  $20 \text{ mm}$                       (d)  $26 \text{ mm}$
7. A shaft of  $50 \text{ mm}$  diameter transmits a torque of  $800 \text{ N-m}$ . The width of the rectangular key used is  $10 \text{ mm}$ . the allowable shear stress of the material of the key being  $40 \text{ MPa}$ , the required length of the key would be
- [ESE - 2016]
- (a)  $60 \text{ mm}$                       (b)  $70 \text{ mm}$   
(c)  $80 \text{ mm}$                       (d)  $90 \text{ mm}$
8. The diameter of the pin in a bushed pin type flexible coupling is to be increased for the purpose of
- [ESE - 2016]
- (a) Higher stress due to shear  
(b) Keeping the magnitude of bending moment small by reducing the unsupported length of the pin

## CHAPTER - 2

### *POWER SCREWS*

#### 2.1 INTRODUCTION

A power screw is a mechanical device used for converting rotary motion into linear motion and transmitting power. Example; screw jack, lead screw of lathe, vice etc.

##### 2.1.1 Advantages

1. Large head capacity for very smaller dimensions of the power screw resulting in compact design.
2. Simple manufacturing and design.
3. Large mechanical advantage for example, load of 15kN can be raised by applying only 400N.
4. Controlled and accurate linear motion.
5. Smooth and noiseless service.
6. A power screw can be designed with self locking property. In screw jack applications, self locking characteristic is required to prevent the load from falling on its own.

##### 2.1.2 Disadvantages

1. Lower efficiency of 40%
2. High friction in threads causes rapid wear of the screw.

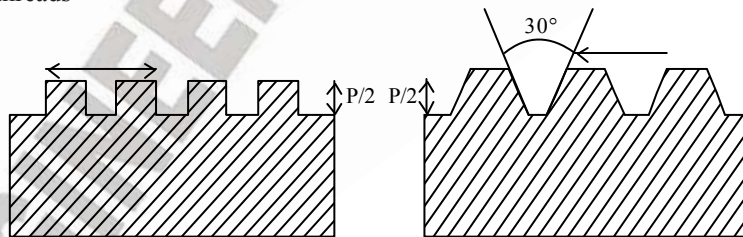
##### 2.1.3 Forms of Threads

1. The threads are used for fastening purpose such as V threads are not suitable for power success. The purpose of fastening threads is to provide high fractional force, which lessens the possibility of loosening the parts assembled by preceded joint.
2. On the other hand, the purpose of power transmission threads is to reduce friction between the screw and nut therefore V threads are not suitable.
3. Screw with smaller angle of thread such as trapezoidal threads are preferred for power transmission.

#### 2.2 TYPES OF POWER SCREW THREADS

There are two mostly used power screw threads are:

1. Square threads
2. Trapezoid threads



##### 2.2.1 Square Threads

###### 2.2.1.1 Advantages

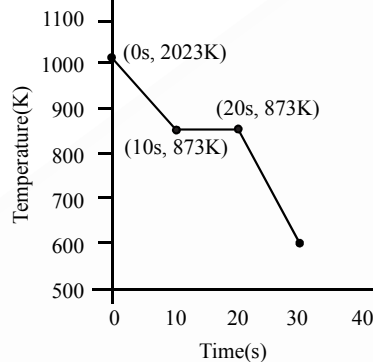
1. Its efficiency is more than trapezoidal threads.
2. There is no radial pressure or side thrust on the nut.

## GATE QUESTIONS

1. Metric thread of 0.8 mm pitch is to be cut on a lathe. Pitch of the lead screw is 1.5mm. If the spindle rotates at 1500 rpm, the speed of rotation of the lead screw (rpm) will be

[GATE - 2017]

2. A hypothetical engineering stress – strain curve shown in the figure has three straight lines PQ, QR, RS with coordinates P(0, 0), Q (0.2, 100), R (0.6, 140) and S (0.8, 130). 'Q' is the yield point, 'r' is the UTS point and 's' the fracture point.

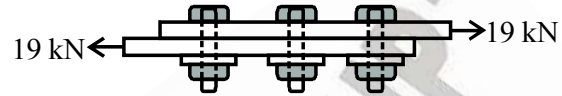


The toughness of the material (in  $\text{MJ/m}^3$ ) is \_\_\_\_\_  
[GATE - 2016]

3. A bolt of major diameter 12 mm is required to clamp two steel plates. Cross sectional area of the threaded portion of the bolt is  $84.3 \text{ mm}^2$ . The length of the threaded portion in grip is 30 mm, while the length of the unthreaded portion in grip is 8 mm. Young's modulus of material is 200 GPa. The effective stiffness (in MN/m) of the bolt in the clamped zone is \_\_\_\_\_.

[GATE - 2014]

4. For the three bolt system shown in the figure, the bolt material has shear yield strength of 200 MPa. For a factor of safety of 2, the minimum metric specification required for the bolt is



[GATE - 2014]

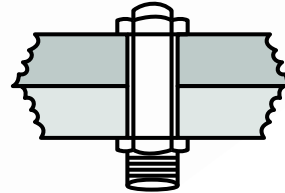
- (a) M 8  
(b) M 10  
(c) M 12  
(d) M 16

5. Two threaded bolts A and B of same material and length are subjected to identical tensile load. If the elastic energy stored in bolt A is 4 times that of the bolt B and the mean diameter of bolt A is 12 mm, the mean diameter of bolt B in mm is

[GATE - 2013]

- (a) 16  
(b) 24  
(c) 36  
(d) 48

6. In a bolted joint two members are connected with an axial tightening force of 2200 N. If the bolt used has metric threads of 4 mm pitch, the torque required for achieving the tightening force is.



[GATE - 2004]

- (a) 0.7 Nm  
(b) 1.0 Nm  
(c) 1.4 Nm  
(d) 2.8 Nm

7. Bolts in the flanged end of pressure vessel are usually pre-tensioned. Indicate which of the following statements is true.

[GATE - 1998]

- (a) Pre-tensioning helps to seal the pressure vessel.  
(b) Pre-tensioning increase the fatigue life of the bolts.  
(c) Pre-tensioning reduces the maximum tensile stress in the bolts.



## CHAPTER - 3

### WELDED JOINTS

#### 3.1 INTRODUCTION

Welding is permanent jointing but un-separable. Riveting is also permanent jointing but separable

##### 3.1.1 Advantages of Welded Joints

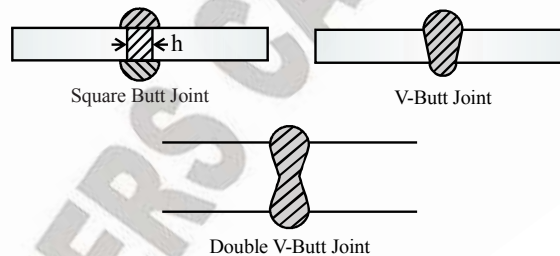
1. Lighter assemblies as compared to riveting where additional cover plates, gussets plates are required
2. Lower cost
3. Changes can be easily made
4. Leak-proof joints
5. Lesser production time
6. Drilling holes in reverted points reduces strength of material.
7. Bad appearance of riveted joints
8. Strength of welded joint is high

##### 3.1.2 Disadvantages

1. Poor vibration damping ability.
2. Thermal distortion due to thermal residual stress therefore stress relieving is a necessity.
3. Quality of weld has to be maintained.

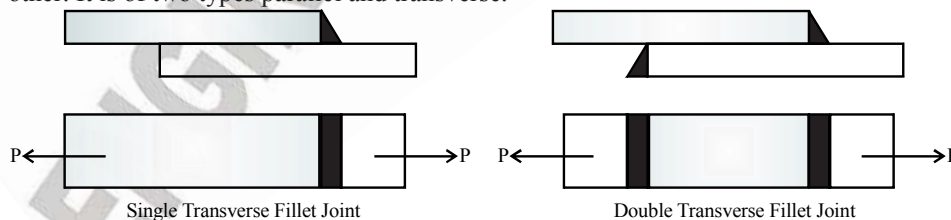
#### 3.2 BUTT JOINTS

A butt joint can be defined as a joint between two components lying approximately in the same plane.



#### 3.3 FILLET JOINTS

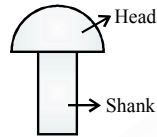
It is also called a lap joint, is a joint between two overlapping plates or components. A fillet weld consists of an approximately triangular cross-section joining two surfaces at right angles to each other. It is of two types parallel and transverse.



## CHAPTER - 4

### *RIVETED JOINTS*

#### 4.1 INTRODUCTION



Rivet is specified by the shank diameter. A 20mm rivet means a rivet having 20mm shank diameter.

##### 4.1.1 Applications of Riveted Joints

1. Riveted joints are used where it is necessary to avoid the thermal after effects of welding.
2. Used for metals with poor weld ability such as aluminum alloys.
3. To join different materials like steel and asbestos.
4. Welded joints have poor resistance to vibrations and impact loads.

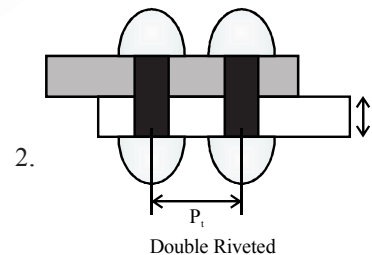
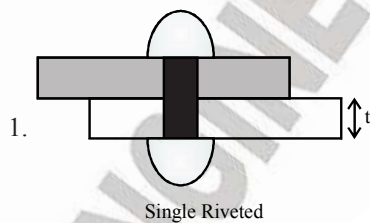
##### 4.1.2 Advantages of Riveted Joint over Welded Joints

1. More reliable in case of vibration and impact loads.
2. Quality of riveted joint can be easily checked.
3. Can be dismantled work without much damaged parent material.

##### 4.1.3 Disadvantages of Riveted Joints Compared to Welded Joint

1. More material cost, holes required for rivets weaker the plate and it is necessary to increase plate thickness to compensate this loss.
2. More labor cost and less productive process.
3. More weight of riveted joints due to overlapping straps requirement.
4. Noisy process
5. Strep concentration is there near holes in plates.

#### 4.2 TYPES OF RIVETED JOINTS



## CHAPTER - 5

### *FRICITION CLUTCHES*

#### 5.1 CLUTCH

It is a mechanical device, which is used to connect or disconnect the source of power from the remaining parts of the power transmission system at the will of operator.

##### 5.1.1 Classification of Clutches

1. **Positive Contact Clutches:** They include square jaw clutches, spiral jaw clutches and toothed clutches. Power transmission is achieved by means of interlocking of jaws or teeth. No slip is there.
2. **Friction Clutches:** They include single and multi plate clutches, cone clutches and centrifugal clutches. Power transmission is achieved by means of friction between contacting surfaces.
3. **Electromagnetic Clutches:** They include magnetic particle clutches, magnetic hysteresis clutches and eddy current clutches. Power transmission is achieved by means of magnetic field.
4. **Fluid Clutches and Couplings:** Power transmission is achieved by means of hydraulic pressure.

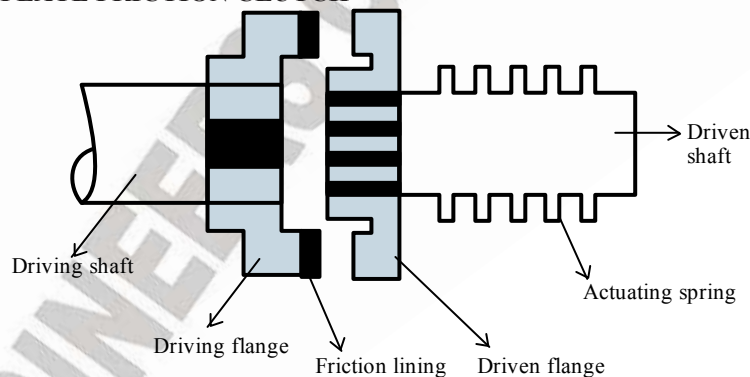
##### 5.1.2 Advantages of Jaw Clutches

1. No slip and engagement is positive
2. No heat is generated during engagement or disengagement.

##### 5.1.3 Disadvantages

1. It can be engaged only when both shafts are stationary or rotate with very small speed difference.
2. It cannot be engaged at high speeds

#### 5.2 SINGLE PLATE FRICTION CLUTCH



1. One flange is rigidly hanged to the driving shaft, while the other is connected to the driven shaft by means of splines. The splines permit free axial movement of the driven flange with respect to driven flange shaft.
2. This axial movement is necessary for engagement and disengagement of the clutch.
3. The actually force is provided by a helical spring which forces the driven flange to move towards driving flange.
4. Power is then transmitted from driving flange to driven flange by means of frictional force.

**GATE QUESTIONS**

1. Single - plate clutch has a friction disc with inner and outer of 20 mm and 40mm, respectively. The friction lining in the disc is made in such a way that the coefficient of friction  $\mu$  varies radially as  $\mu = 0.01 r$ , where  $r$  is in mm. The clutch needs to transmit a friction torque of 18.85 kN-mm. As per uniform pressure theory, the pressure (in MPa) on the disc is \_\_\_\_\_  
[GATE - 2017]
2. A disc clutch with a single friction surface has coefficient of friction equal to 0.3. The maximum pressure which can be imposed on the friction material is 1.5 MPa. The outer diameter of the clutch plate is 200 mm and its internal diameter is 100 mm. Assuming uniform wear theory for the clutch plate, the maximum torque (in N.m) that can be transmitted is \_\_\_\_\_  
[GATE - 2014]
3. A clutch has outer and inner diameter 100 mm and 40 mm respectively. Assuming a uniform pressure of 2 MPa and coefficient of friction of liner material 0.4, the torque carrying capacity of the clutch is \_\_\_\_\_  
[GATE - 2008]
- (a) 148 Nm (b) 196 Nm  
(c) 372 Nm (d) 490 Nm
4. A disk clutch is required to transmit 5 kW at 2000 rpm. The disk has a friction lining with coefficient of friction equal to 0.25. Bore radius of friction lining is equal to 25 mm. Assume uniform contact pressure of 1 MPa. The value of outside radius of the friction lining is \_\_\_\_\_  
[GATE - 2006]
- (a) 39.4 mm (b) 49.5 mm  
(c) 97.9 mm (d) 142.9 mm
5. Axial operation claw clutches having self-locking tooth profile.  
[GATE - 1987]
- (a) Can be disengaged at any speed  
(b) Can be disengaged only unloaded  
(c) Can be engaged only when unloaded  
(d) Can work only with load.

## CHAPTER - 6

### BRAKES

#### 6.1 BRAKES

A brake is a mechanical device, which is used to absorb energy passed by a moving system or mechanism by means of friction.

Brake capacity depends upon the following three factors.

1. The frictional force between braking surfaces.
2. The contacting area of braking surfaces.
3. Radius of brake drum
4.  $\mu$
5. Ability of the brake to dissipate heat that is equivalent to the energy being absorbed.

#### 6.2 ENERGY EQUATIONS

Consider a mechanical system of mass  $m$ , moving with velocity  $V_1$  is slowed down to velocity  $V_2$ ,

$\therefore$  During the period of braking, the  $KE = \frac{1}{2}m(V_1^2 - V_2^2)$

Similarly for a rotating body,  $KE = \frac{1}{2}I(\omega_1^2 - \omega_2^2)$

$$KE = \frac{1}{2}mk^2(\omega_1^2 - \omega_2^2)$$

Where  $k$  is radius of gyration

In certain applications, like hoists, the brake absorbs the potential energy released by the moving weight during the braking period.

$$PE = mgh$$

Depending upon the type of applications, the total energy absorbed by the brake is determined by

$$E = T \times \theta$$

Where  $\theta$  is angle through which brake drum rotates during the braking period (rad)

**Example.** A solid CI disk, 1m in diameter and 0.2m thick is used as flywheel. It is rotating at 350rpm. It is brought to rest in 1.5s by means of a brake calculate

- (a) The energy absorbed by the brake
- (b) The torque capacity of the brake  $P_a = 7200\text{kg/m}^3$

**Solution.**

$$D = 1\text{m}, t = 0.2\text{m}, N_1 = 350\text{rpm}, N_2 = 0$$

$$t = 1.5\text{s}$$

$$(a) E = \frac{1}{2}mk^2(\omega_1^2 - \omega_2^2)$$

$$\omega_1 = \frac{2\pi(350)}{60} = 36.63\text{rad/sec}$$

$$m = (\pi r^2 h) (7200)$$

$$m = \pi(.5)^2 \times (.2) (7200) = 1130.97\text{kg}$$

$$k = \frac{d}{\sqrt{8}} \quad (\text{for solid disk about its axis of rotation})$$

$$k = \frac{1}{\sqrt{8}} \Rightarrow k^2 = \frac{1}{8}$$

**CHAPTER - 7**  
**BELTS****7.1 BELT DRIVES**

Belt, chain and rope drives are called flexible drives.

Gear drives are rigid drives.

Belts are used to transmit power between two shafts by means of friction.

**7.1.1 Advantages of Belt Drives**

1. Operation is smooth and silent
2. It can transmit power over considerable distance between the axes of driving and driven shafts.
3. They can transmit only a definite load, which if exceeded, will cause the belt to slip over the pulley.
4. It has ability to absorb shocks and damp vibration
5. It has low cost and simple design.

**7.1.2 Disadvantages of Belt Drives**

1. It has large dimensions and occupies more space.
2. The VR is not constant due to belt slip.
3. It has low efficiency
4. It has short life

Two types of cross section

- (i) Flat belt                      (ii) V-belt

**7.1.3 Advantages of Flat Belts Over V-Belts**

1. Relatively cheap and easy to maintain
2. Their maintenance consists of periodic adjustment in the centre distance between shafts in order to compensate stretching.
3. Different VR can be obtained by using a stepped pulley, where the belt is shifts from one step to another, having different diameter.
4. Simple and inexpensive
5. Can be used for long distances up to 15m
6. Efficiency of flat belt is more than efficiency of V-belt

**7.1.4 Disadvantages of Flat Belt Drives Over V-Belt Drives**

1. The power transmitting capacity of flat belt is low.
2. VR is less than V-belt
3. Flat belts are noisier than V-belts
4. Only horizontal and not vertical.

**7.1.5 Advantages of V-Belts**

1. Force of friction between the surfaces of the belt and v-grooved pulley is high due to wedge action. This wedging action permits a smaller arc of contact, increases the pulling capacity of the belt and consequently results in increase in power transmitting capacity.
2. Shorter distance belts
3. High VR up to 7: 1
4. Smooth operation



## ESE OBJ QUESTIONS

**1. Assertion (A):** In chain drives, angle of articulation through which link rotates during engagement and disengagement, is greater for a small number of teeth.

**Reason (R):** The greater angle of articulation will increase the life of the chain.

[ESE - 2015]

- (a) Both A and R are true and R is the correct explanation of A  
 (b) Both A and R are true but R is not a correct explanation of A  
 (c) A is true but R is false.  
 (d) A is false but R is true.

**2.** If the velocity ratio  $\sigma$  for an open belt drive is 8 and the speed of driving pulley is 800 r.p.m, then considering an elastic creep of 2% the speed of the driven pulley is

[ESE - 2015]

- (a) 104.04 r.p.m.                      (b) 102.04 r.p.m.  
 (c) 100.04 r.p.m.                      (d) 98.04 r.p.m.

**3.** If the angle of warp on smaller pulley of diameter 250 mm is  $120^\circ$  and diameter of larger pulley is twice the diameter of smaller pulley, then the center distance between the pulleys for an open belt drive is

[ESE - 2015]

- (a) 1000 mm                              (b) 750 mm  
 (c) 500 mm                                (d) 250 mm

**4.** If  $T_1$  and  $m$  represent the maximum tension and mass per unit length of a belt, the maximum permissible speed of the belt is given by

[ESE - 2014]

- (a)  $\sqrt{\frac{T_1}{3m}}$                                       (b)  $\sqrt{\frac{3T_1}{m}}$   
 (c)  $\sqrt{\frac{2T_1}{3m}}$                                       (d)  $\sqrt{\frac{T_1}{m}}$

**5.** Which of the following statements are correct regarding power transmission through V-belts?

1. V-belts are used at the high-speed end.

2. V-belts are used at the low-speed end.

3. V-belts are standard lengths.

4. V-angles of pulleys and belts are standardized.

Select the correct answer using the code given below:

[ESE - 2014]

- (a) 1 and 3 only                              (b) 2 and 4 only  
 (c) 2, 3 and 4                                (d) 1, 3 and 4

**6. Statement (I):** In short open-belt drives, an idler pulley is used in order to increase the angle of contact on the smaller pulley for higher power transmission.

**Statement (II):** The idler pulley facilitates changing the speed of the driven shaft, while the main or driven shaft runs at constant speed.

[ESE - 2014]

- (a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).  
 (b) Both Statement (I) and Statement (II) are individually true but Statement (II) is NOT the correct explanation of Statement (I).  
 (c) Statement (I) is true but Statement (II) is false.  
 (d) Statement (I) is false but Statement (II) is true.

**7.** Considering the effect of centrifugal tension in a flat drive with  $T_1$  = tight side tension and  $T_c$  = centrifugal tension and  $m$  = mass per unit length of belt, the velocity of the belt for maximum power transmission is given by:

[ESE - 2013]

- (a)  $V = \sqrt{\frac{T_1}{3m}}$                                       (b)  $V = \sqrt{\frac{T_c}{3m}}$   
 (c)  $V = \sqrt{\frac{(T_1 - T_c)}{3m}}$                                       (d)  $V = \sqrt{\frac{(T_1 + T_c)}{3m}}$

**8. Statement (I):** Generally, for larger size pulleys, curved arms are used.

## CHAPTER - 8

### CHAIN DRIVES

#### 8.1 INTRODUCTION

A chain can be defined as a series of links connected by pin joints. It has some features of belt drive and some of gear drive.

##### 8.1.1 Advantages

1. It can be used for long as well as short distance range.
2. Number of shafts can be driven
3. Small overall dimensions
4. Positive drive and has no slip
5. High efficiency (96% to 98%).
6. No initial tension required
7. Easy to replace.

##### 8.1.2 Disadvantages

1. More wear.
2. Less précised motion
3. Noisy operation

#### 8.2 DESIGN OF SPUR GEARS

A mechanical drive is defined as a mechanism, which is intended to transmit mechanical power over a certain distance, usually involving a change in speed and torque.

Two groups of mechanical drives are

1. Mechanical drives that transmit power by means of friction e.g. belt and rope drive.
2. Mechanical drives that transmit power by means of engagement e.g. chain drive and gear drive.



1. Selection of a proper mechanical drive for a given application depends upon number of factors such as centre distance, VR, shifting arrangement, maintenance and cost.
2. Gear drive is a positive drive and has constant speed.

#### 8.3 CHAIN

The belt drive is not a positive drive because of creep and slip. The chain drive is a positive drive. Like belts, chains can be used for larger centre distances. They are made of metal and due to this chain is heavier than the belt but they are flexible like belts. It also requires lubrication from time to time. The lubricant prevents chain from rusting and reduces wear.

The chain and chain drive are shown in figure. The sprockets are used in place of pulleys. The projected teeth of sprockets fit in the recesses of the chain. The distance between roller centers of two adjacent links is known as pitch. The circle passing through the pitch centers is called pitch circle.

## CHAPTER - 9

### GEARS

#### 9.1 GEAR DRIVES

Gears are defined as toothed wheels or multi lobed comes, which transmit power and motion from one shaft to another by means of successive engagement of teeth.

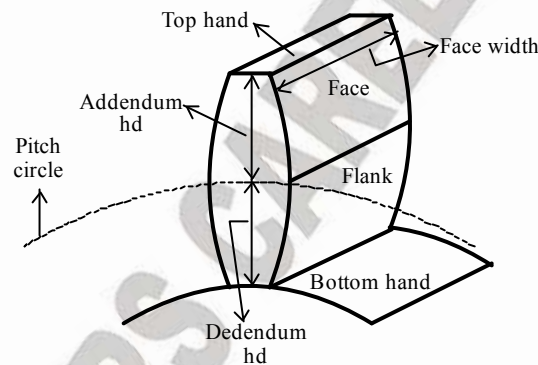
##### 9.1.1 Advantages

1. It is a positive drive with constant VR
2. CD between shafts is small therefore compact construction
3. It can transmit very large power, even beyond the range of chain and belt drive
4. It can transmit motion at very low velocity which is not possible with belt drives
5. 99% efficiency
6. Provision of gear shifting is there in gear boxes.

##### 9.1.2 Disadvantages

1. Gear drives are costly and their maintenance cost is also higher.
2. Precise alignment is also required.

#### 9.2 TERMINOLOGY



1. **Pinion:** smaller of the two mating gear
2. **Gear:** larger of the two rotating gear
3. **Pitch circle:** Pitch circle is the curve of intersection of the pitch surface of revolution and the plane of rotation. It is an imaginary circle that rotates without slipping with the pitch circle of a mating gear corresponding diameter is pitch circle dia. (PCD)
4. **Addendum ( $h_a$ ):** height of tooth above PCD
5. **Dedendum ( $h_d$ ):** height of tooth below PCD
6. **Clearance (C):** Clearance is the amount by which dedendum of a given gear exceeds the addendum of its mating tooth.
7. **Face width (b):** It is width of tooth measured parallel to axis.
8. **Tooth space:** The width of the space between two adjacent teeth measured along the pitch circle is called the tooth space.
9. **Working depth:** Sum of addendum of gear is engagement.
10. **C.D:** It is the distance between centres of pitch circles of mating gears.
11. **Pressure angle:** It is the angle which the line of action makes with the common tangent to the pitch circles. The pressure angle is also called angle of obliquity.

## GATE QUESTIONS

1. A spur pinion of pitch diameter 50 mm rotates at 200 rad/s and transmits 3 kW power. The pressure angle of the tooth of the pinion is  $20^\circ$ . Assuming that only one pair of the teeth is in contact, the total force (in newton) exerted by a tooth of the pinion on the tooth on a mating gear is \_\_\_\_\_.

[GATE - 2014]

2. A pair of spur gears with module 5 mm and a center distance of 450 mm is used for a speed reduction of 5: 1. The number of teeth on pinion is \_\_\_\_\_.

[GATE - 2014]

3. Which one of the following is used to convert a rotational motion into a translational motion?

[GATE - 2014]

- (a) Bevel gears
- (b) Double helical gears
- (c) Worm gears
- (d) Rack and pinion gears

4. For the given statements:

I. Mating spur gear teeth is an example of higher pair.

II. A revolute joint is an example of lower pair.

Indicate the correct answer.

[GATE - 2014]

- (a) Both I and II are false
- (b) I is true and II is false
- (c) I is false and II is true
- (d) Both I and II are true.

5. Two cutting tools are being compared for machining operation. The tool life equation are:

Carbide tool:  $VT^{1.6} = 3000$

HSS tool:  $VT^{0.6} = 200$

Where V is the cutting speed in m/min and T is the tool life in min. The carbide tool will provide higher tool life if the cutting speed in m/min exceeds.

[GATE - 2013]

- (a) 15.0
- (b) 39.4
- (c) 49.3
- (d) 60.0

**Linked Statement for Q.6 & Q.7**

A  $20^\circ$  full depth involute spur pinion of 4 mm module and 21 teeth is to transmit 15 kW at 960 rpm. It face width is 25 mm.

6. The tangential force transmitted (in N) is

[GATE - 2009]

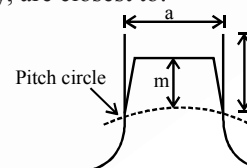
- (a) 3552
- (b) 2611
- (c) 1776
- (d) 1305

7. Given that the tooth geometry factor is 0.32 and the combined effect dynamic load and allied factors intensifying the stress is 1.5; the minimum allowable stress (in MPa) for the gear material is

[GATE - 2009]

- (a) 242.0
- (b) 166.5
- (c) 121.0
- (d) 74.0

8. One tooth of a gear having 4 module and 32 teeth is shown in the figure. Assume that the gear tooth and the corresponding tooth space make equal intercepts on the pitch circumference. The dimensions 'a' and 'b', respectively, are closest to:



[GATE - 2008]

- (a) 6.08 mm, 4 mm
- (b) 6.48 mm, 4.2 mm
- (c) 6.28 mm, 4.3 mm
- (d) 6.28 mm, 4.1 mm

9. Match the type of gears with their most appropriate description.

**Type of gear**

- A. Helical
- B. Spiral Bevel
- C. Hypoid
- D. Rack and pinion

**Description**

- (i) Axes non parallel and non intersecting
- (ii) Axes parallel and teeth are inclined to the axis

## CHAPTER - 10

### BEARING

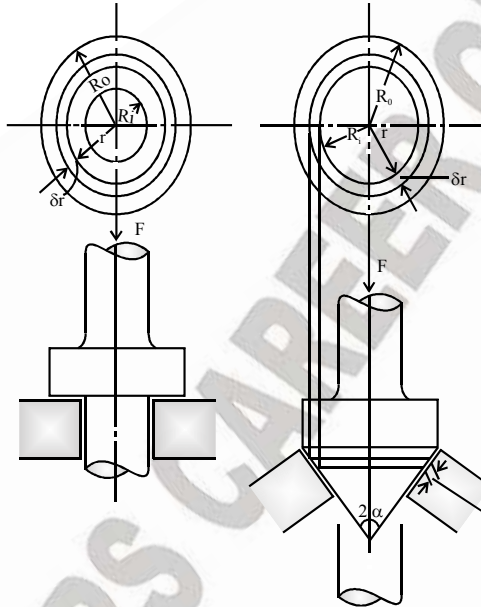
#### 10.1 INTRODUCTION

When a rotating shaft is subjected to an axial load, the thrust (axial force) is taken either by a pivot or a collar. Examples are the shaft of a steam turbine and propeller shaft of a ship.

##### 10.1.1 Collar Bearing

A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface.

The surface of the collar may be plane (flat) normal to the shaft (**Fig.**) of conical shape (**Fig.**).



##### 10.1.2 Pivot Bearing

When the axial load is taken by the end of the shaft which is inserted in a recess to bear the thrust, it is called a *pivot bearing* or simply a *pivot*. It is also known as *footstep bearing*.

$$\begin{aligned} &= \int_{R_i}^{R_o} p \times 2\pi r dr = \int_{R_i}^{R_o} \frac{C}{r} \times 2\pi r dr \\ &= \int_{R_i}^{R_o} 2\pi C dr = (2\pi C r)_{R_i}^{R_o} = 2\pi C (R_o - R_i) = 2\pi p r (R_o - R_i) \end{aligned}$$

or pressure intensity  $p$  at a radius  $r$  of the collar,

$$p = \frac{F}{2\pi r (R_o - R_i)}$$

In a flat pivot, in which  $R_i = 0$ , the pressure would be infinity at the centre of the bearing ( $r = 0$ ), which cannot be true. Thus, the uniform wear theory has a flaw in it.

Collars and pivots, using the above two theories, have been analysed below:







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## CHAPTER - 1

### MECHANICS

#### 1.1 FORCE

Force may be defined as a push or pull which produces or tends to produce a change in the state of rest or of uniform motion of a body or change in the direction of motion of the body.

$$F = ma$$

Force = mass  $\times$  acceleration

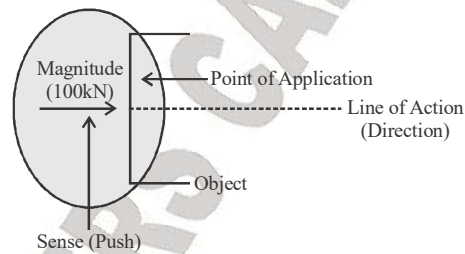
This is the fundamental equation of motion which gives the measurement of force. It is a vector equation.

##### 1.1.1 Effects of a Force

1. Change the motion
2. Change the direction
3. Change the size or shape
4. Give rise to internal stresses

##### 1.1.2 Characteristics of Force

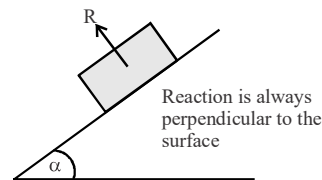
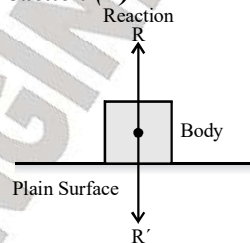
1. The magnitude of the force in known units such as Newton's or kilonewtons. (i.e., 50 N, 100 KN etc.)
2. The line of action or direction of the force may be taken with respect to reference lines.
3. Its point of application, that is the point on the body at which forces acts.
4. Sense or nature of the force (Push or Pull)



##### 1.1.3 Classification of Forces

###### 1. According to the Nature of Force

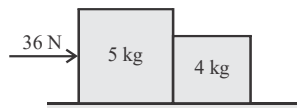
###### (i) Action ( $R$ ) and Reaction ( $R'$ )





# GATE QUESTIONS

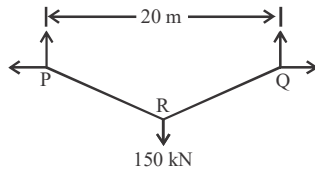
1. Two rigid bodies of mass 5 kg and 4 kg are at rest on a frictionless surface until acted upon by a force of 36 N as shown in the figure. The contact force generated between the two bodies is



[GATE - 2018]

- (a) 4.0 N                      (b) 7.2 N  
(c) 9.0 N                      (d) 16.0 N

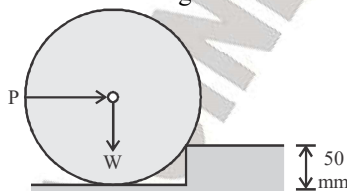
2. A Cable PQ of length 25 m is supported at two ends at the same level as shown in the figure. The horizontal distance between the supports is 20 m. A point load of 150 kN is applied at point R which divides it into two equal parts.



Neglecting the self-weight of the cable, the tension (in kN, integer value) in the cable due to the applied load will be \_\_\_\_\_

[GATE - 2018]

3. A cylinder of radius 250 mm and weight,  $W = 10$  kN is rolled up an obstacle of height 50 mm by applying a horizontal force  $P$  at its centre as shown in the figure.



[GATE - 2018]

All interfaces are assumed frictionless. The minimum value of  $P$  is

- (a) 4.5 kN                      (b) 5.0 kN  
(c) 6.0 kN                      (d) 7.5 kN

4. An aircraft approaches the threshold of a runway strip at a speed of 200 km/h. The pilot decelerates the aircraft at a rate of  $1.697 \text{ m/s}^2$  and takes 18 s to exit the runway strip. If the deceleration after exiting the runway is  $1 \text{ m/s}^2$ , then the distance (in m, up to one decimal place) of the gate position from the location of exit on the runway is \_\_\_\_\_

[GATE - 2018]

5. Two disks A and B with identical mass ( $m$ ) and radius ( $R$ ) are initially at rest. They roll down from the top of identical inclined planes without slipping. Disk A has all of its mass concentrated at the rim, while Disk B has its mass uniformly distributed. At the bottom of the plane, the ratio of velocity of the center of disk A to the velocity of the center of disk B is

[GATE - 2017]

- (a)  $\sqrt{\frac{3}{4}}$                       (b)  $\sqrt{\frac{3}{2}}$   
(c) 1                              (d)  $\sqrt{2}$

6. A particle of unit mass is moving on a plane. Its trajectory, in polar coordinates, is given by  $r(t) = t^2$ ,  $\theta(t) = t$ , where  $t$  is time. The kinetic energy of the particle at time  $t = 2$  is

[GATE - 2017]

- (a) 4                              (b) 12  
(c) 16                              (d) 24

7. The magnitudes of vectors  $P$ ,  $Q$  and  $R$  are 100 kN, 250 kN and 150 kN, respectively as shown in the figure.

## CHAPTER - 2

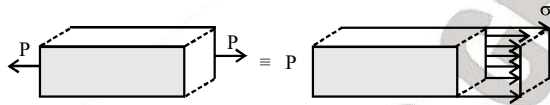
### STRESS AND STRAIN

#### 2.1 STRESS ( $\sigma$ )

When a material is subjected to an external force, a resisting force is set up within the component. The internal resistance force per unit area acting on a material or intensity of the forces distributed over a given section is called the stress at a point.

(i) It uses original cross section area of the specimen and also known as engineering stress or conventional stress.

$$\text{Therefore, } \sigma = \frac{P}{A}$$



Where P is expressed in Newton (N) and A, original area, in square meters (m), the stress  $\sigma$  will be expressed in  $\text{N/m}^2$ . This unit is called Pascal (Pa).

(ii) As Pascal is a small quantity, in practice, multiples of this unit is used.

$$1\text{kPa} = 10^3 \text{ Pa} = 10^3 \text{ N/m}^2$$

(kPa = Kilo Pascal)

$$1\text{MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2 = 1\text{N/mm}^2$$

(MPa = Mega Pascal)

$$1\text{GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2$$

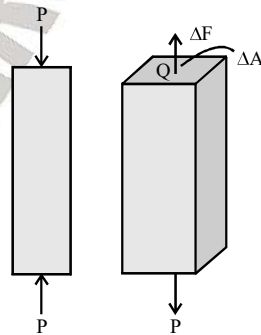
(GPa = Giga Pascal)

Let us take an example: A rod 10 mm  $\times$  10 mm cross-section is carrying an axial tensile load 10 kN. In this rod the tensile stress developed is given by

$$(\sigma_1) = \frac{P}{A} = \frac{10\text{kN}}{(10\text{mm} \times 10\text{mm})} = \frac{10 \times 10^3 \text{ N}}{100\text{mm}^2} = 100\text{N/mm}^2 = 100\text{MPa}$$

The resultant of the internal forces for an axially loaded member is normal to a section cut perpendicular to the member axis.

The force intensity on the shown section is defined as the normal stress.



$$\sigma = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} \quad \text{and} \quad \sigma_{\text{avg}} = \frac{P}{A}$$

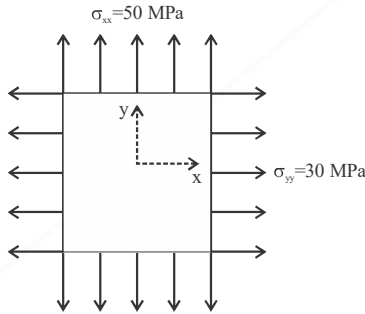
**GATE QUESTIONS**

1. The deformation in concrete due to sustained loading is

[GATE - 2018]

- (a) Creep
- (b) Hydration
- (c) Segregation
- (d) Shrinkage

2. A plate in equilibrium is subjected to uniform stresses along its edges with magnitude  $\sigma_{xx} = 30$  MPa and  $\sigma_{yy} = 50$  MPa as shown in the figure



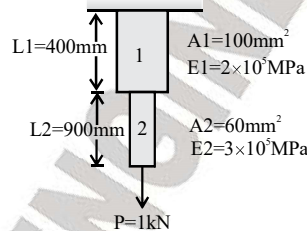
The Young's modulus of the material is  $2 \times 10^{11}$  N/m<sup>2</sup> and the Poisson's ratio is 0.3. If  $\sigma_{zz}$  is negligibly small and assumed to be zero, then the strain  $\epsilon_{zz}$  is

[GATE - 2018]

- (a)  $-120 \times 10^{-6}$
- (b)  $-60 \times 10^{-6}$
- (c) 0.0
- (d)  $120 \times 10^{-6}$

3. Consider the stepped bar made with a linear elastic material and subjected to an axial load of 1 kN, as shown in the figure.

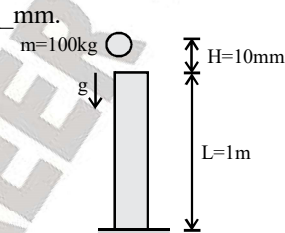
[GATE - 2017]



Segments 1 and 2 have cross-sectional area of 100 mm<sup>2</sup> and 60 mm<sup>2</sup>, Young's modulus of

$2 \times 10^5$  MPa and  $3 \times 10^5$  MPa, and length of 400 mm and 900 mm, respectively. The strain energy (in N-mm, up to one decimal place) in the bar due to the axial load is \_\_\_\_\_.

4. A point mass of 100 kg is dropped onto a massless elastic bar (cross-sectional area = 100 mm<sup>2</sup>, length = 1 m, Young's modulus = 100 GPa) from a height H of 10 mm as shown (Figure is not to scale). If  $g = 10$  m/s<sup>2</sup>, the maximum compression of the elastic bar is \_\_\_\_\_ mm.



5. A horizontal bar, fixed at one end ( $x = 0$ ), has a length of 1m, and cross-sectional area of 100mm<sup>2</sup>. Its elastic modulus varies along its length as given by  $E(x) = 100 e^{-x}$  GPa, where  $x$  is the length coordinate (in m) along the axis of the bar. An axial tensile load of 10 kN is applied at the free end ( $x = 1$ ). The axial displacement of the free end is \_\_\_\_\_ mm.

[GATE - 2017]

6. An initially stress-free massless elastic beam of length L and circular cross-section with diameter d ( $d \ll L$ ) is held fixed between two walls as shown. The beam material has Young's modulus E and coefficient of thermal expansion  $\alpha$ .



## ESE CONV QUESTIONS

1. Derive equations for compressive and tensile thermal stresses.

[ME ESE - 2015]

**Solution.**

Compressive thermal stress for completely constrained bar

Let a bar is rigidly fixed between two rigid supports A and B.

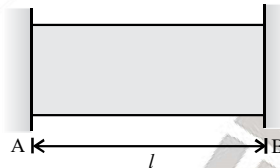
$\alpha$  is thermal expansion coefficient

$\Delta T$  is increase in temperature

$E$  is young's modulus of bar

$l$  is length of bar

$A$  is Cross-sectional area of bar

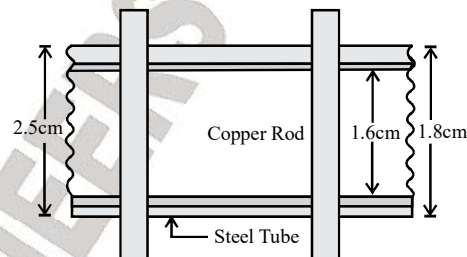


As temperature of bar is increased it will thermally elongate but completely restricted by rigid supports producing equal and opposite reaction at A and B to maintain equilibrium.

2. A steel tube 2.5 cm external diameter and 1.8 cm internal diameter encloses a copper rod 1.6 cm diameter to which it is rigidly joined at each end. If, at a temperature of  $20^\circ\text{C}$  there is no longitudinal stress, calculate the stresses in the rod and tube when the temperature is raised to  $210^\circ\text{C}$ . Given:  $E_s = 210 \text{ Pa}$  and  $\alpha_s = 12 \times 10^{-6}/^\circ\text{C}$  and  $E_C = 100 \text{ GPa}$  and  $\alpha_C = 20 \times 10^{-6}/^\circ\text{C}$ .

[ME ESE - 2014]

**Solution.**



Let  $l$  is the length of the copper bar and steel tube.

Difference in free expansion =  $\alpha_c \cdot l \cdot t - \alpha_s \cdot l \cdot t = (\alpha_c - \alpha_s) l t = (20 - 12) \times 190 \times 10^{-6} l$

$= 1.52 \times 10^{-3} l$

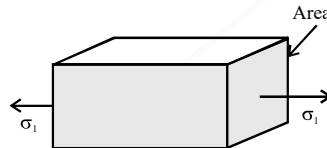
Net effect in length due to compressive force  $P$  on the copper bar and tensile force  $P$  on steel tube.

$$\therefore \Delta l = \left( \frac{P l}{A E} \right)_{\text{copper}} + \left( \frac{P l}{A E} \right)_{\text{steel}}$$

**CHAPTER - 3****PRINCIPAL STRESS & STRAIN****3.1 STATES OF STRESS****3.1.1 Uni-Axial Stress**

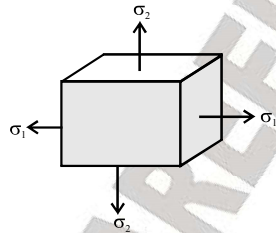
Only one non-zero principal stress, i.e.  $\sigma_1$ .

Right side figure represents Uni-axial state of stress.

**3.1.2 Bi-Axial Stress**

One principal stress equals zero, two do not, i.e.  $\sigma_1 > \sigma_3$ ;  $\sigma_2 = 0$

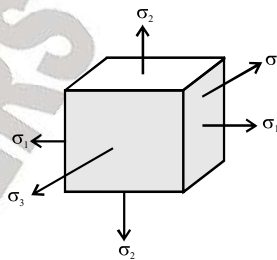
Right side figure represents Bi-axial state of stress.

**3.1.3 Tri-Axial Stress**

Three non-zero principal stresses,

i.e.  $\sigma_1 > \sigma_2 > \sigma_3$

Right side figure represents Tri axial state of stress.

**3.1.4 Isotropic Stress**

Three principal stresses

are equal, i.e.  $\sigma_1 = \sigma_2 = \sigma_3$

Right side figure represents Isotropic state of stress.

**GATE QUESTIONS**

1. A soil sample is subjected to a hydrostatic pressure,  $\sigma$ . The Mohr circle for any point in the soil sample would be

[GATE - 2017]

- (a) A circle of radius  $\sigma$  and center at the origin
- (b) A circle of radius  $\sigma$  and center at a distance  $\sigma$  from the origin
- (c) A point at a distance  $\sigma$  from the origin
- (d) A circle of diameter  $\sigma$  and center at the origin

2. A rod of length 20mm is stretched to make a rod of length 40mm. Subsequently, it is compressed to make a rod of final length 10mm. consider the longitudinal tensile strain as positive and compressive strain as negative. The total true longitudinal strain in the rod is

[GATE - 2017]

- (a) -0.5
- (b) -0.69
- (c) -0.75
- (d) -1.0

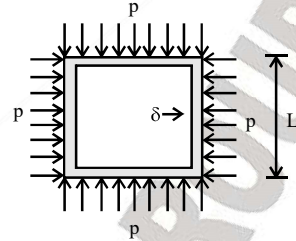
3. The state of stress at a point is  $\sigma_x = \sigma_y = \sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$  and  $\tau_{xy} = \tau_{yx} = 50\text{Mpa}$ . The maximum normal stress (in MPa) at that point is \_\_\_\_\_

[GATE - 2017]

4. A rectangular region in a solid is in a state of plane strain. The (x, y) coordinates of the corners of the undeformed rectangle are given by P(0, 0), Q (4, 0), R (4, 3), S (0, 3). The rectangle is subjected to uniform strain,  $\epsilon_{xx} = 0.001$ ,  $\epsilon_{yy} = 0.002$ ,  $\gamma_{xy} = 0.003$ . The deformed length of the elongated diagonal, upto three decimal places, is \_\_\_\_\_ unit.

[GATE - 2017]

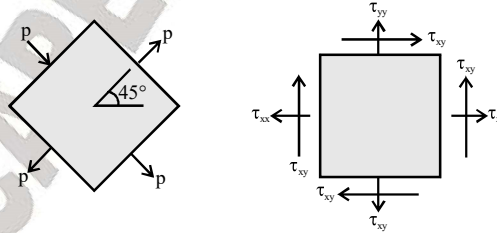
5. A square plate of dimension  $L \times L$  is subjected to a uniform pressure load  $p = 250$  MPa on its edges as shown in the figure. Assume plane stress conditions. The Young's modulus  $E = 200$  GPa.



The deformed shape is a square of dimension  $L - 2\delta$ . If  $L = 2$  m and  $\delta = 0.001$  m, the Poisson's ratio of the plate material is \_\_\_\_\_.

[GATE - 2016]

6. The state of stress at a point on an element is shown in figure (a). The same state of stress is shown in another coordinate system in figure (b).



The components ( $\tau_{xx}$ ,  $\tau_{yy}$ ,  $\tau_{xy}$ ) are given by

[GATE - 2016]

- (a)  $(P/\sqrt{2}, -P/\sqrt{2}, 0)$
- (b)  $(0, 0, P)$
- (c)  $(P, -P, -P/\sqrt{2})$
- (d)  $(0, 0, P/\sqrt{2})$

7. In a structural member under fatigue loading, the minimum and maximum stresses developed at the critical point are 50 MPa and 150 MPa, respectively. The endurance, yield, and the ultimate strengths of the material are 200 MPa, 300 MPa, and 400 MPa, respectively. The factor of safety using modified Goodman criterion is

[GATE - 2016]

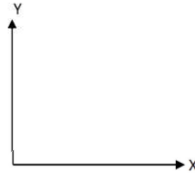
- (a)  $\frac{3}{2}$
- (b)  $\frac{8}{5}$



**CHAPTER - 4*****BENDING MOMENT AND SHEAR FORCE DIAGRAM*****4.1 SHEAR FORCE AND BENDING MOMENT**

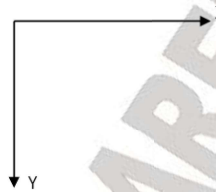
At first we try to understand what shear force is and what is bending moment?

We will not introduce any other co-ordinate system. We use general co-ordinate axis as shown in the figure.



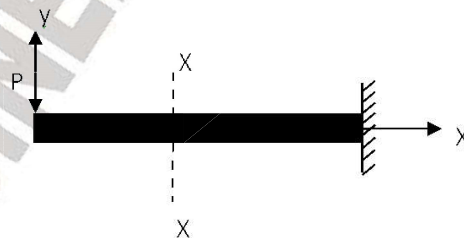
This system will be followed in shear force and bending moment diagram and in deflection of beam. Here downward direction will be negative i.e. negative Y-axis. Therefore downward deflection of the beam will be treated as negative.

Some books fix a co-ordinate axis as shown in the following figure.



Here downward direction will be positive i.e. positive Y-axis. Therefore downward deflection of the beam will be treated as positive. As beam is generally deflected in downward directions and this co-ordinate system treats downward deflection as positive deflection.

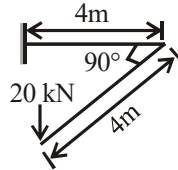
Consider a cantilever beam as shown subjected to external load 'P'. If we imagine this beam to be cut by a section X-X, we see that the applied force tends to displace the left-hand portion of the beam relative to the right-hand portion, which is fixed in the wall. This tendency is resisted by internal forces between the two parts of the beam. At the cut section a resistance shear force ( $V_x$ ) and a bending moment ( $M_x$ ) is induced. This resistance shear force and the bending moment at the cut section is shown in the left hand and right hand portion of the cut beam.



Using the three equations of equilibrium.

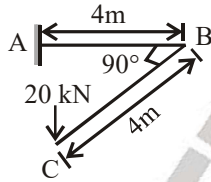
**ESE CONV QUESTIONS**

1. Draw the bending moment and torsional moment diagram for the beam as shown. The load is perpendicular to the beam plane.

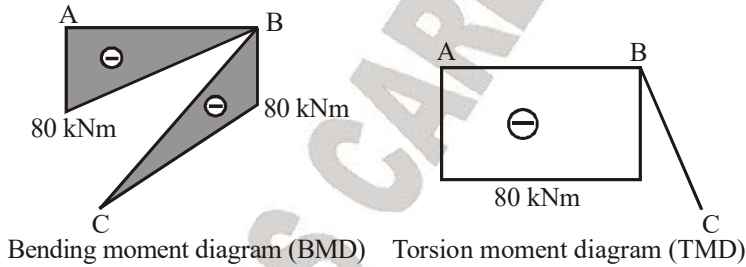


[CE ESE - 2015]

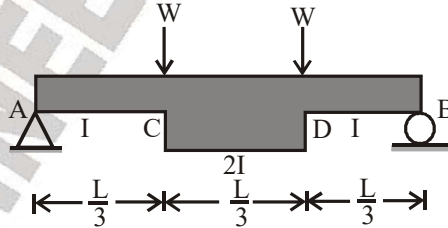
**Solution.**



There will be torsion in beam BC.



2. A simply supported beam carries two point loads  $W$  each at its one-third sections as shown in figure. Determine the maximum deflection at its mid span and slope at an end using the conjugate beam method



[CE ESE - 2014]

**Solution.**

Reactions in real beam:  
By symmetry:  $R_A = R_B = W$

## CHAPTER - 5

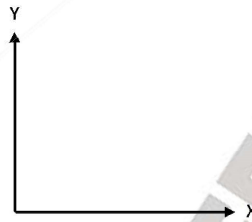
### *DEFLECTION OF BEAM*

#### 5.1 INTRODUCTION

1. We know that the axis of a beam deflects from its initial position under action of applied forces.
2. In this chapter we will learn how to determine the elastic deflections of a beam.

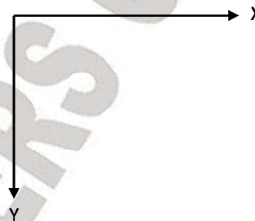
##### 5.1.1 Selection of Co-ordinate Axis

We will not introduce any other co-ordinate system. We use general co-ordinate axis as shown in the figure. This system will be followed in deflection of beam and in shear force and bending moment diagram. Here downward direction will be negative i.e. negative Y-axis. Therefore downward deflection of the beam subjected to a given loading where we will use the formula,



$$EI \frac{d^2y}{dx^2} = M_x.$$

Some books fix a co-ordinate axis as shown in the following figure. Here downward direction will be positive i.e. positive Y-axis. Therefore downward deflection of the beam will be treated as positive. As beam is generally deflected in downward directions and this coordinate system treats downward deflection is positive deflection.



To determine the value of deflection of beam subjected to a given loading where we will use the

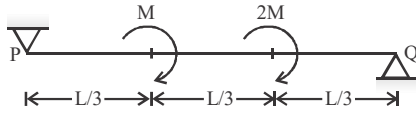
formula,  $EI \frac{d^2y}{dx^2} = -M_x.$

##### 5.1.2 Why to calculate the deflections

1. To prevent cracking of attached brittle materials
2. To make sure the structure not deflected severely and to “appear” safe for its occupants.
3. To help analyzing statically indeterminate structures.
4. Information on deformation characteristics of members is essential in the study of vibrations of machines

**GATE QUESTIONS**

1. The figure shows a simply supported beam PQ of uniform flexural rigidity EI carrying two moments M and 2M

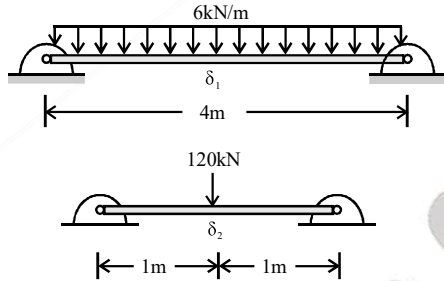


[GATE - 2018]

The slope at P will be

- (a) 0
- (b)  $ML/(9EI)$
- (c)  $ML/(6EI)$
- (d)  $ML/(3EI)$

2. Two prismatic beams having the same flexural rigidity of  $1000 \text{ kN-m}^2$  are shown in the figures.



If the mid-span deflections of these beams are denoted by  $\delta_1$  and  $\delta_2$  (as indicated in the figures), the correct option is

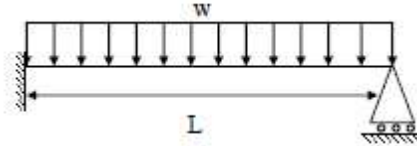
[GATE - 2017]

- (a)  $\delta_1 = \delta_2$
- (b)  $\delta_1 < \delta_2$
- (c)  $\delta_1 > \delta_2$
- (d)  $\delta_1 \gg \delta_2$

3. A simply supported rectangular concrete beam of span 8m has to be span 8m has to be prestressed with a force of 1600 kN. The tendon is of parabolic profile having zero eccentricity at the supports. The beam has to carry an external uniformly distributed load of intensity 30 kN/m. Neglecting the self-weight of the beam, the maximum dip (in meters, up to two decimal places) of the tendon at the mid-span to balance the external load should be \_\_\_\_\_

[GATE - 2017]

4. A beam of length L is carrying a uniformly distributed load w per unit length. The flexural rigidity of the beam is EI. The reaction at the simple support at the right end is



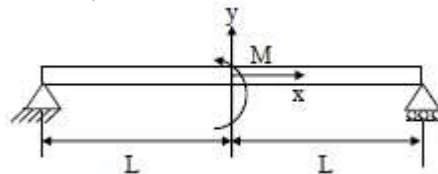
[GATE - 2016]

- (a)  $\frac{wL}{2}$
- (b)  $\frac{3wL}{8}$
- (c)  $\frac{wL}{4}$
- (d)  $\frac{wL}{8}$

5. A simply supported beam of length 2L is subjected to a moment M at the mid-point  $x=0$  as shown in the figure. The deflection in the domain  $0 \leq x \leq L$  is given by

$$W = \frac{-Mx}{12EI} (L-x)(x+c)$$

Where E is the Young's modulus, I is the area moment of inertia and c is a constant (to be determined).



The slope at the center  $x=0$  is

[GATE - 2016]

- (a)  $ML/(2EI)$
- (b)  $ML/(3EI)$
- (c)  $ML/(6EI)$
- (d)  $ML/(12EI)$

6. A 3 m long simply supported beam of uniform cross section is subjected to a uniformly distributed load of  $w = 20 \text{ kN/m}$  in the central 1 m as shown in the figure.

## CHAPTER - 6

### BENDING & SHEAR STRESSES

#### 6.1 EULER'S BERNOULLI'S EQUATION or (Bending Stress Formula) or Bending Equation

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

Where  $\sigma$  is bending Stress

M is bending moment

I is moment of inertia

E is modulus of elasticity

R is radius of curvature

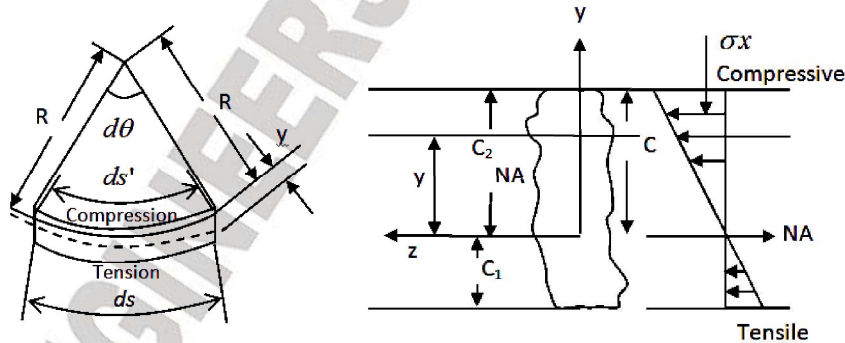
y is distance of the fibre from NA (Neutral axis)

#### 6.2 ASSUMPTIONS IN SIMPLE BENDING THEORY

All of the foregoing theory has been developed for the case of pure bending i.e. constant B.M. along the length of the beam. In such case

1. The shear force at each c/s is zero.
2. Normal stress due to bending is only produced.
3. Beams are initially straight.
4. The material is homogenous and isotropic i.e. it has a uniform composition and its mechanical properties are the same in all directions.
5. The stress strain relationship is linear and elastic.
6. Young's Modulus is the same in tension as in compression.
7. Sections are symmetrical about the plane of bending.
8. Sections which are plane before bending remain plane after bending.

6.3  $\sigma_{\max} = \sigma_t = \frac{Mc_1}{I}$  ;  $\sigma_{\min} = \sigma_c = \frac{Mc_2}{I}$  (Minimum in sense of sign)



#### 6.4 SECTION MODULUS (Z)

$$Z = \frac{I}{y}$$



## ESE OBJ QUESTIONS

1. A steel wire of 20mm diameter is bent into a circular shape of 10m radius. If E, the modulus of elasticity, is  $2 \times 10^6 \text{ kg/cm}^2$ , then the maximum tensile stress induced in the wire is, nearly

[CE ESE - 2018]

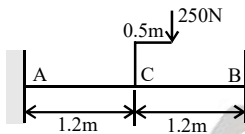
- (a)  $2 \times 10^3 \text{ kg/cm}^2$       (b)  $4 \times 10^3 \text{ kg/cm}^2$   
 (c)  $2 \times 10^4 \text{ kg/cm}^2$       (d)  $4 \times 10^4 \text{ kg/cm}^2$

2. A long rod of uniform rectangular section with thickness t, originally straight, is bent into the form of a circular arch with displacement d at the mid-point of span l. The displacement d may be regarded as small as compared to the length l. The longitudinal surface strain is

[CE ESE - 2018]

- (a)  $\frac{2td}{l^2}$       (b)  $\frac{4td}{l^2}$   
 (c)  $\frac{8td}{l^2}$       (d)  $\frac{16td}{l^2}$

3. In fig



A horizontal bar of 40 mm diameter solid section is 2.40 m long and is rigidly held at both ends so that no angular rotation occurs axially or circumferentially at the ends (as shown in figure). The maximum tensile stress in the bar is nearly

[CE ESE - 2018]

- (a)  $12.2 \text{ N/mm}^2$       (b)  $13.7 \text{ N/mm}^2$   
 (c)  $15.2 \text{ N/mm}^2$       (d)  $16.7 \text{ N/mm}^2$

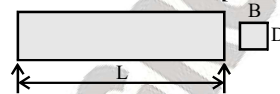
4. In the case of a rectangular beam subjected to a transverse shearing force, the ratio of maximum shear stress to average shear stress is

[CE ESE - 2018]

- (a) 0.75      (b) 1.00  
 (c) 1.25      (d) 1.50

5. A homogenous prismatic simply supported beam is subjected to point load F. The load can be placed anywhere along the span of the beam. The very maximum flexural stress developed in the beam is

[CE ESE - 2017]



- (a)  $\frac{3FL}{2BD^2}$       (b)  $\frac{3FL}{4BD^2}$   
 (c)  $\frac{2FL}{3BD^2}$       (d)  $\frac{4FL}{3BD^2}$

6. The span of a cantilever beam is 2m. The cross-section of the beam is a hollow square with external sides 10mm; and its  $I = 4 \times 10^5 \text{ mm}^4$ . The safe bending stress for the beam material is  $100 \text{ N/mm}^2$ . The safe concentrated load at the free end would be

[CE ESE - 2017]

- (a) 100 N      (b) 200 N  
 (c) 300 N      (d) 400 N

7. Consider the following statements:

- The shear stress distribution across the section of a circular shaft subjected to twisting varies parabolically.
- The shear stress at the centre of a circular shaft under twisting moment is zero
- The shear stress at the extreme fibres of a circular shaft under twisting moment is maximum.

Which of the above statements is/are correct?

[CE ESE - 2017]

- (a) 1, 2 and 3      (b) 1 only  
 (c) 2 only      (d) 3 only

8. **Assertion (A):** The failure surface of an axially loaded mild steel tension specimen of circular cross-section is along a plane at  $45^\circ$  to the axis of the specimen.

## CHAPTER - 7

### TORSION, THIN & THICK CYLINDER

#### 7.1 INTRODUCTION

1. In machinery, the general term “shaft” refers to a member, usually of circular cross-section, which supports gears, sprockets, wheels, rotors, etc. and which is subjected to torsion and to transverse or axial loads acting singly or in combination.

2. An “axle” is a non-rotating member that supports wheels, pulleys, ... and carries no torque.

3. A “spindle” is a short shaft. Terms such as line shaft, head shaft, stub shaft, transmission shaft, countershaft, and flexible shaft are names associated with special usage.

#### 7.2 TORSION OF CIRCULAR SHAFTS

1. Equation for shafts subjected to torsion “T”

$$\text{Torsion Equation, } \frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$$

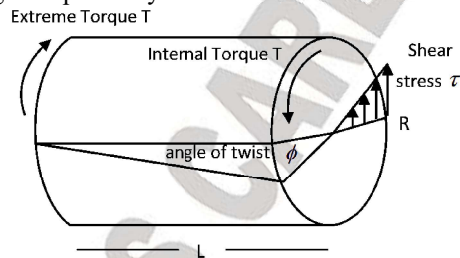
Where  $j$  is polar moment of inertia

$\tau$  is shear stress induced due to torsion  $T$

$G$  is modulus of rigidity

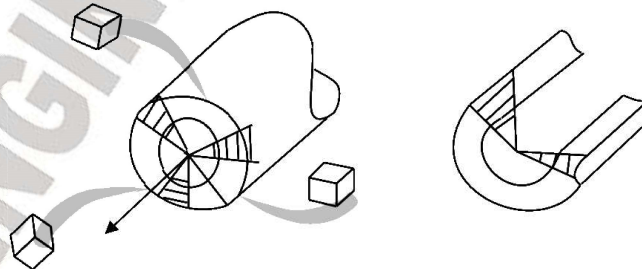
$\theta$  is angular deflection or shaft

$R, L$  is shaft radius and length respectively.



##### 7.2.1 Assumptions

1. The bar is acted upon by a pure torque
2. The section under consideration is remote from the point of application of the load and from a change in diameter.
3. Adjacent cross sections originally plane and parallel remain and parallel after twisting, and any radial line remains straight.
4. The material obeys Hooke's law.
5. Cross-sections rotate as if rigid, i.e. every diameter rotates through the same angle.



## ESE OBJ QUESTIONS

1. A solid shaft A of diameter  $D$  and length  $l$  is subjected to a torque  $T$ ; another shaft B of the same material and of the same length, but half the diameter, is also subjected to the same torque  $T$ . The ratio between the angles of twist of shaft B to that of shaft A is

[CE ESE - 2018]

- (a) 32 (b) 16  
(c) 8 (d) 4

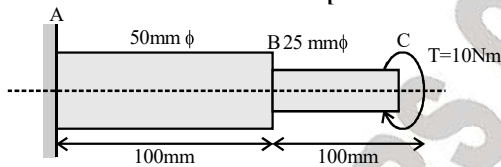
2. The required diameter for a solid shaft to transmit 400 kW at 150 rpm, with the working shear stress not to exceed  $80 \text{ MN/m}^2$ , is nearly.

[CE ESE - 2018]

- (a) 125 mm (b) 121 mm  
(c) 117 mm (d) 113 mm

3. A stepped steel shaft is subjected to a clockwise torque of 10 Nm at its free end. Shear modulus of steel is 80 GPa. The strain energy stored in the shaft is

[CE ESE - 2017]



- (a) 1.73 N mm (b) 2.52 Nmm  
(c) 3.46 N mm (d) 4.12 N mm

4. A solid shaft transmits 150kW at a shear stress of 70MPa running at a frequency of 3Hz. What will be the shear stress when the frequency is 1.5Hz?

[CE ESE - 2015]

- (a) 35Mpa (b) 50MPa  
(c) 57MPa (d) 140MPa

5. A hollow circular shaft has the diameters 50cm and 30cm and is subjected to a torque. If the realized maximum shear stress is  $30 \text{ N/mm}^2$ ,

what is the applied torque to nearest units

[CE ESE - 2015]

- (a) 160Nm (b) 320Nm  
(c) 80Nm (d) 32Nm

6. What is the Polar modulus of a solid circular metal shaft of diameter 8cm ?

[CE ESE - 2015]

- (a)  $64\pi \text{ cm}^3$  (b)  $32\pi \text{ cm}^3$   
(c)  $16\pi \text{ cm}^3$  (d)  $8\pi \text{ cm}^3$

7. What is the power transmitted by a 100mm diameter solid shaft at 150rpm without exceeding a maximum stress of  $60 \text{ N/mm}^2$ ?

[Take  $\pi^2 = 10$ ]

[CE ESE - 2015]

- (a) 187.5kW (b) 18.75kW  
(c) 1.875kW (d) 1875kW

8. A metal shaft of solid circular section rotates at 160rpm and is subjected to a torque of 1500Nm. What is the power in kW, transmitted by the shaft ?

[CE ESE - 2015]

- (a)  $32\pi$  (b)  $16\pi$   
(c)  $12\pi$  (d)  $8\pi$

9. What is the diameter  $d$  of a solid circular shaft when subjected to a torque  $T$  with a corresponding maximum shear stress of magnitude  $f_s$ ?

[CE ESE - 2015]

- (a)  $\frac{16T}{\pi f_s}$  (b)  $\frac{\pi f_s}{16T}$   
(c)  $\sqrt{\frac{16T}{\pi f_s}}$  (d)  $3\sqrt{\frac{16T}{\pi f_s}}$

10. A hollow shaft of 16mm outside diameter and 12mm inside diameter is subjected to a torque of 40N-m. The shear stresses at the

## CHAPTER - 8

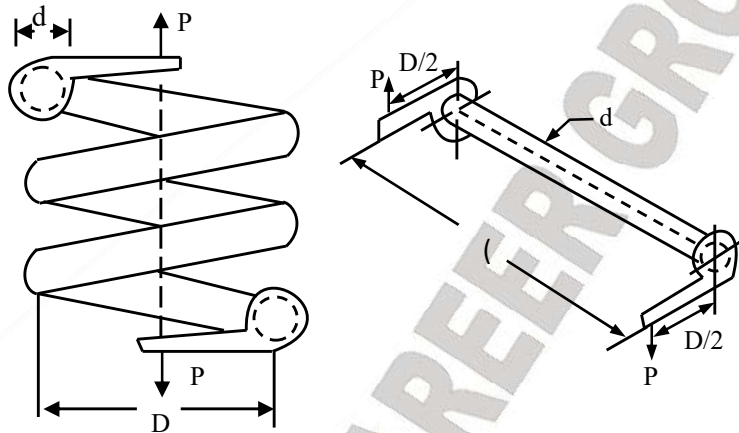
### SPRINGS

#### 8.1 INTRODUCTION

A spring is a mechanical device which is used for the efficient storage for the efficient and release of energy.

#### 8.2 HELICAL SPRING-STRESS EQUATION

Let us a close-coiled helical spring has coil diameter  $D$ , wire diameter  $d$  and number of turn  $n$ .



The spring material has a shearing modulus  $G$ . The spring index,  $C = \frac{D}{d}$ . If a force 'P' is exerted in both ends as shown.

The work done by the axial force 'P' is converted into strain energy and stored in the spring.

$$U = (\text{average torque}) \times (\text{angular displacement}) = \frac{T}{2} \times \theta$$

$$\text{From the figure we get, } \theta = \frac{TL}{GJ}$$

$$\text{Torque, } T = \frac{PD}{2}$$

$$\text{Length of wire, } L = \pi Dn$$

$$\text{Polar moment of inertia, } (J) = \frac{\pi d^4}{32}$$

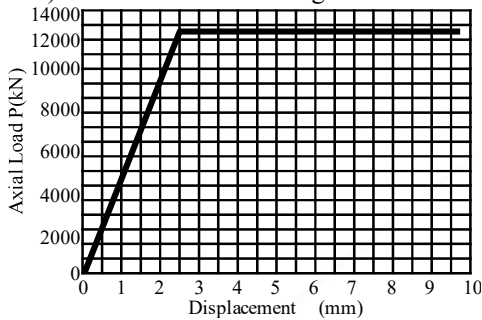
$$\text{Therefore } U = \frac{4P^2 D^3 n}{Gd^4}$$

Accordingly to Castiglano's theorem, the displacement corresponding to force  $P$  is obtained by partially differentiating strain energy with respect to that force.

$$\text{Therefore, } \delta = \frac{\delta U}{\delta P} = \frac{\delta}{\delta P} \left[ \frac{4P^2 D^3 n}{Gd^4} \right] = \frac{8PD^3 n}{Gd^4}$$

## GATE QUESTIONS

1. A 2m long, axially loaded mild steel rod of 8mm diameter exhibits the load - displacement ( $P - \delta$ ) behavior as shown in figure.



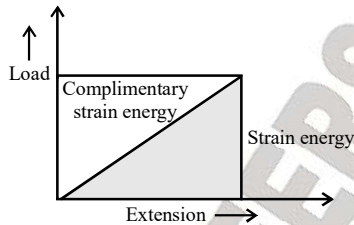
Assume the yield stress of steel as 250 MPa. The complementary strain energy (in N-mm) stored in the bar up to its linear elastic behaviour will be \_\_\_\_

[GATE - 2017]

## SOLUTIONS

**Sol. 1. (15,707.963)**

Complimentary strain energy for linear materials.



Complimentary strain energy = strain energy

$$\begin{aligned}
 &= \frac{1}{2} \sigma_y \times \epsilon \times A \times L \\
 &= 12 \times 250 \times \frac{2.5}{2000} \times \frac{\pi}{4} \times 8^2 \times 2000 \\
 &\left( \because \epsilon = \frac{\delta \ell}{\ell} \right) \\
 &= 15,707.963 \text{ N-mm}
 \end{aligned}$$

**CHAPTER - 9*****THEORY OF COLOUMN & STRAIN ENERGY*****9.1. INTRODUCTION**

- 1.Strut: A member of structure which carries an axial compressive load.
- 2.Column: If the strut is vertical it is known as column.
- 3.A long, slender column becomes unstable when its axial compressive load reaches a value called the critical buckling load.
- 4.If a beam element is under a compressive load and its length is an order of magnitude larger than either of its other dimensions such a beam is called as columns.
- 5.Due to its size its axial displacement is going to be very small compared to its lateral deflection called buckling.
- 6.Buckling does not vary linearly with load it occurs suddenly and is therefore dangerous
- 7.Slenderness Ratio: The ratio between the length and least radius of gyration.
- 8.Elastic Buckling: Buckling with no permanent deformation
- 9.Euler's buckling is only valid for long, slender objects in the elastic region.
- 10.For short columns, a different set of equations must be used.

**9.2. WHICH IS THE CRITICAL LOAD?**

1. At this value the structure is in equilibrium regardless of the magnitude of the angle (provided it stays small)
2. Critical load is the only load for which the structure will be in equilibrium in the disturbed position
3. At this value, restoring effect of the moment in the spring matches the buckling effect of the axial load represents the boundary between the stable and unstable conditions.
4. If the axial load is less than  $P_{cr}$  the effect of the moment in the spring dominates and the structure returns to the vertical position after a small disturbance-stable condition.
5. If the axial load is larger than  $P_{cr}$  the effect of the axial force predominates and the structure buckles-unstable condition.
6. Because of the large deflection caused by buckling, the least moment of inertia can be expressed as,  $I = Ak^2$
7. Where  $A$  is the cross sectional area and  $r$  is the radius of gyration of the cross sectional area, i.e.

$$k_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

8. Note that the smallest radius of gyration of the column, i.e. the least moment of inertia  $I$  should be taken in order to find the critical stress.  $L/k$  is called the slenderness ratio, it is a measure of the column's flexibility.

**9.3. EULER'S CRITICAL LOAD FOR LONG COLUMN****9.3.1 Assumption**

1. The column is perfectly straight and of uniform cross-section
2. The material is homogenous and isotropic
3. The material behaves elastically
4. The load is perfectly axial and passes through the Centroid of the column section.
5. The weight of the column is neglected.



**CHAPTER - 10*****THEORIES OF FAILURE*****10.1 INTRODUCTION**

When some external load is applied on a body, the stresses and strains are produced in the body. The stresses are directly proportional to the strains within the elastic limit. This means when the load is removed, the body will return to its original shape. There is no permanent deformation in the body.

However, If the stress produced in the body due to the application of the load, is beyond the elastic limit, the permanent deformations occur in the body. This means if the load is removed, the body will not retain its original shape. There are some permanent deformations in the body. Whenever permanent deformations occur in the body, the body is said to have “failed”. This should be clear that failure does not mean rupture of the body.

The failure takes place when a certain limiting value is reached by one of following:

1. The maximum principal stress
2. The maximum principal strain.
3. The maximum shear stress.
4. The maximum strain energy
5. The maximum shear strain energy.

In all the above cases,

$\sigma_1, \sigma_2, \sigma_3$  = principal stresses in any complex system

$\sigma^*$  = tensile or compressive stress at the elastic limit.

**1. Maximum Principal Stress Theory**

According to this theory, the failure of a material will occur when the maximum principal tensile stress ( $\sigma_1$ ) in the complex system reaches the value of the maximum stress at the elastic limit in simple tension or the minimum principal stress (i.e, the maximum principal compressive stress) reaches the value of the maximum stress at the elastic limit in simple compression .

Let in a complex three dimensional stress system.

$\sigma_1, \sigma_2$  and  $\sigma_3$  = principal stresses at a point in three perpendicular directions. The stresses  $\sigma_1$  and  $\sigma_2$  are tensile and  $\sigma_3$  is compressive. Also  $\sigma_1$  is more than  $\sigma_2$  .

$\sigma_t^*$  = tensile stress at elastic limit in simple tension.

$\sigma_c^*$  = compressive stress at elastic limit in simple compression.

Then according to this theory, the failure will take place if

$$\sigma_1 \geq \sigma_t^* \text{ in simple tension} \quad \dots(1)$$

$$\text{Or } |\sigma_3| \geq \sigma_c^* \text{ in simple compression} \quad \dots(1.1)$$

Where  $|\sigma_3|$  represents the absolute value of  $\sigma_3$  .

This is the simplest and oldest theory of failure and is known as Rankine’s theory. If the maximum principal stress ( $\sigma_1$ ) is the design criterion, then maximum principal stress must not exceed the permissible stress ( $\sigma_t$ ) for the given material.

$$\text{Hence, } \sigma_1 = \sigma_t \quad \dots(1.2)$$

Where  $\sigma_t$  = permissible stress and given by

$$\sigma_t = \sigma_t^* / \text{safety factor.}$$

# **ESE**

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# **2019**

## **THEORY OF MACHINES**

**MECHANICAL ENGINEERING**



**ECG**  
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## CHAPTER - 1

### *BASIC CONCEPT*

#### 1.1. INTRODUCTION

Theory of machine is a study of relative motion between various parts of Machine and forces acting upon them. It can be further divided into the two parts.

##### 1.1.1 Kinematics

Kinematics is study of motion without considering external force acting on the machine parts.

$$\text{Velocity, } v = \frac{d\vec{s}}{dt}$$

$$\text{Acceleration, } a = \frac{d\vec{v}}{dt}$$

$$J = \frac{d\vec{a}}{dt}$$

##### 1.1.2 Dynamics

Dynamics is study of motion with effect of some external force acting on the machine parts. It can further divide into two parts

###### 1. Statics

It is a study of forces and their effects on machine and its parts, when it is at rest.

###### 2. Kinematics

It is study of combined effect of mass & motion of Machine and its parts

$$\text{Momentum } \vec{F}_{\text{ext}} = \frac{d}{dt}(m\vec{v})$$

#### 1.2 KINEMATIC LINK OR ELEMENT

It is a part of machine which moves relative with respect to some other part of machine. It should be a resistant body so that it is capable of transmitting the motion from one part to other part of machine with negligible deformation.

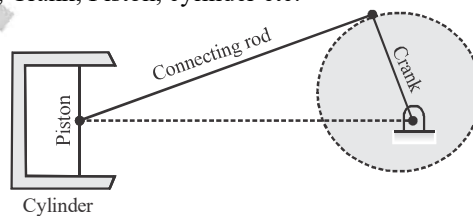
#### 1.3 CLASSIFICATION OF LINKS

The detailed classification of links is described below:

##### 1.3.1. Rigid Link

It is a link which transmits required motion & force without any significant deformation.

**Example.** Connecting rod, Crank, Piston, cylinder etc.



**Slider Crank Chain Mechanism**



# IAS OBJ QUESTIONS

1. For one degree of freedom planar mechanism having 6 links, which one of the following is the possible combination?

[IAS - 2007]

- (a) Four binary links and two ternary links
- (b) Four ternary links and two binary links
- (c) Three ternary links and three binary links
- (d) One ternary link and five binary links

2. Consider the following statements in respect of four bar mechanism:

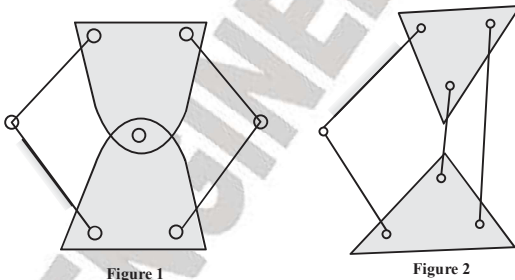
1. It is possible to have the length of one link greater than the sum of length of the other three links.
2. If the sum of the lengths of the shortest and the longest links is less than the sum of lengths of the other two, it is known as the Grashoff's linkage.
3. It is possible to have the sum of the lengths of the shortest and the longest links greater than that of the remaining two links.

Which of these statements is/are correct?

[IAS - 2003]

- (a) 1, 2 and 3
- (b) 2 and 3
- (c) 2 only
- (d) 3 only

3. **Assertion (A):** The kinematic mechanism shown in figure 1 and figure 2 below are the kinematic inversion of the same kinematic chain.

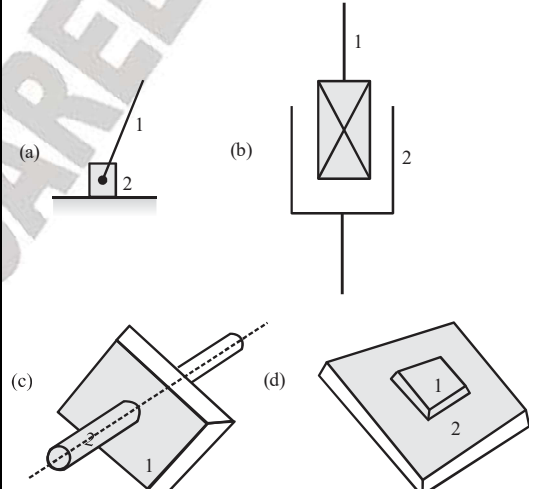


**Reason (R):** Both the kinematic mechanisms have equal number of links and revolute joints, but different fixed links.

[IAS - 2002]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not a correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

4. Which one of the following "Kinematic pairs" has 3 degrees of freedom between the pairing elements?



[IAS - 2002]

5. In a four-link kinematic chain, the relation between the number of links (L) and number of pairs (j) is

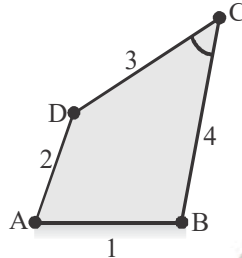
[IAS - 2000]

- (a)  $L = 2j + 4$
- (b)  $L = 2j - 4$
- (c)  $L = 4j + 2$
- (d)  $L = 4j - 2$

6. The given figure shows a/an

**CHAPTER - 2*****INVERSION OF KINEMATIC CHAINS*****2.1 INTRODUCTION**

The method of obtaining different mechanism by fixing different links in a kinematic chain is known as inversion of mechanism. Through process of inversion the relative motion between various link is not changed in any manner.

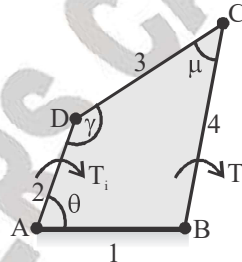
**2.2 FOUR BAR CHAIN OR QUADRATIC CYCLE CHAIN**

As name suggested that, the chain is combination of four kinematic pairs i.e. four Revolute Pair such that the relative motion between the links is completely constrained.

The link AD which is adjacent to the link AB or frame (which is usually fixed) is called as driver (crank) and link BC to which motion is transferred is known as follower (rocker) and link DC which transmits motion from link AD to link BC is known as coupler (connecting rod).

**2.2.1 Mechanical Advantage (M.A)**

It is the ratio of the output force or torque to the input force or torque at any instant.



From Principle of conservation of energy, we know that:

Power Input = Power output

If  $T_i$  is torque applied by input link 2 with angular speed  $\omega_i$  and  $T_o$  is torque obtained by the output link 4 with angular speed  $\omega_o$  respectively.

Then,  $T_i \omega_i = T_o \omega_o$

$$(\therefore P = T \omega)$$

$$M.A = \frac{T_o}{T_i} = \frac{\omega_i}{\omega_o}$$

To achieve M.A. to be infinite, the angular velocity  $\omega_o$  of the output link 4 becomes zero at extreme positions.

Two extreme condition can be obtained at  $\gamma = 0^\circ$  or  $\gamma = 180^\circ$ . In both conditions the link 2 and link 3 will be in the same line.

## ESE OBJ QUESTIONS

1. Consider the following motions :

1. Piston reciprocating inside an engine cylinder
  2. Motion of a shaft between foot-step bearings
- Which of the above can rightly be considered as successfully constrained motion?

[ESE - 2016]

- (a) 1 only                                      (b) 2 only  
(c) Both 1 and 2                              (d) Neither 1 nor 2

2. In a crank and slotted lever type quick return mechanism, the link moves with an angular velocity of 20 rad/s, while the slider moves with a linear velocity of 1.5 m/s. The magnitude and direction of Coriolis component of acceleration with respect to angular velocity are

[ESE - 2015]

- (a)  $30 \text{ m/s}^2$  and direction is such as to rotate slider velocity in the same sense as the angular velocity  
(b)  $30 \text{ m/s}^2$  and direction is such as to rotate slider velocity in the opposite sense as the angular velocity  
(c)  $60 \text{ m/s}^2$  and direction is such as to rotate slider velocity in the same sense as the angular velocity  
(d)  $60 \text{ m/s}^2$  and direction is such as to rotate slider velocity in the opposite sense as the angular velocity.

3. In a crank and slotted lever quick-return motion, the distance between the fixed centres is 150 mm and the length of the driving crank is 75 mm. The ratio of the time taken on the cutting and return strokes is

[ESE - 2014]

- (a) 1.5                                              (b) 2.0  
(c) 2.2                                              (d) 2.93

4. Which one of the following mechanisms is an inversion of double slider-crank chain?

[ESE - 2014]

- (a) Elliptic trammels  
(b) Beam engine  
(c) Oscillating cylinder engine  
(d) Coupling rod of a locomotive

5. In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 160 mm and the driving crank is 80 mm long. The ratio of the taken by cutting and return strokes is

[ESE - 2012]

- (a) 0.5                                              (b) 1  
(c) 1.5                                              (d) 2

6. In an elliptic trammel, the length of the link connecting the two sliders is 100mm. the tracing pen is placed on 150 mm extension of this link. The major and minor axes of the ellipse traced by the mechanism would be

[ESE - 2012]

- (a) 250 mm and 150 mm  
(b) 250 mm and 100 mm  
(c) 500 mm and 300 mm  
(d) 500 mm and 200 mm

7. **Statement (I):** Method of obtaining different mechanisms by fixing in turn different links in a kinematic chain is known as inversion.

**Statement (II):** Scotch Yoke mechanism is an inversion of a double slider crank mechanism.

[ESE - 2012]

- (a) Both Statements (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)  
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)  
(c) Statement (I) is true but Statement (II) is false  
(d) Statement (I) is false but Statement (II) is true.

**CHAPTER - 3*****KINEMATICS OF MACHINE*****3.1 INTRODUCTION**

Kinematics deals with study of relative motion between the various parts of machine. The motion leads to concept of displacement, velocity and acceleration.

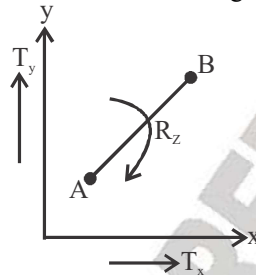
**3.2 VELOCITY IN MECHANISM**

Velocity analysis of a mechanism can be carried out by following methods:

- (i) Relative velocity method
- (ii) Instantaneous centre method

**3.2.1 Relative Velocity Method**

In the planar motion, rigid body has two motion like sliding and rotation as shown in figure

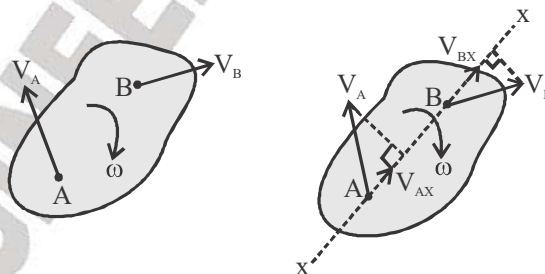


Consider a rigid link AB, which translates in X & Y direction as  $T_x$  &  $T_y$  respectively. Thus, both ends of rigid link translate with same velocity and acceleration along the same path.

Now, consider a link AB, which has rotation along Z direction as  $R_z$ . Thus, both ends of rigid link rotate with same angular velocity and angular acceleration.

**1. Direction of Velocity**

Consider a rigid body rotating with angular speed ( $\omega$ ) and it has two points A & B on it. The velocity of point A & B are  $V_A$  &  $V_B$  respectively shown in the figure. Since, the velocity component  $V_{AX}$  &  $V_{BX}$  parallel to connecting line should be equal if  $V_{AX}$  &  $V_{BX}$  are if velocity component are different then following conditions may arise as:



- (i) If  $V_B$  is greater than  $V_A$ , then, point B elongate the rigid body along  $V_A$  the direction of velocity  $V_{BX}$  which is impossible.

**CHAPTER - 4****DYNAMICS OF MACHINE****4.1 INTRODUCTION**

Dynamic deals with study of forces acting upon the various parts of machine. Dynamic forces are always present when machines working under full or part load condition.

**4.1.1 D'Alembert's Principle**

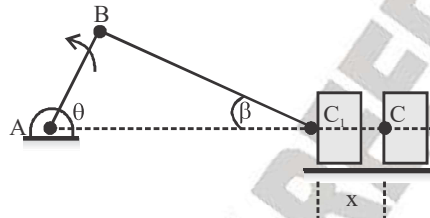
It states that, the inertia force and couples, and the external force and torques on a body together gives statically equilibrium.

Thus,  $\Sigma F + F_i = 0$

and  $\Sigma T + C_i = 0$

**4.2 SLIDER-CRANK MECHANISM**

Let, the crank AB has turned through angle  $\theta$  from inner dead centre (IDC) and slider changes its position C to  $C_1$  with displacement X.

**4.2.1 Displacement of Piston**

It is a distance travel by piston.

$$x = C C_1 = CA - C_1 A$$

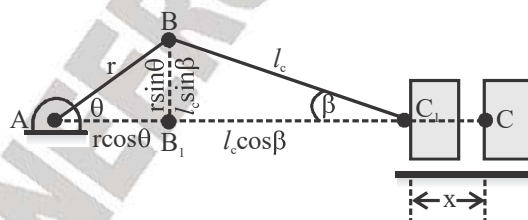
$$x = (l_c + r) - (r \cos \theta + l_c \cos \beta)$$

$$= (nr + r) - (r \cos \theta + nr \cos \beta)$$

$$[\because n = l_c / r]$$

$$= r [n + 1 - \cos \theta - n \cos \beta]$$

← →



Where,

$$\sin^2 \beta + \cos^2 \beta = 1$$

$$\cos \beta = \sqrt{1 - \sin^2 \beta}$$

$$\text{At } BB_1, r \sin \theta = l_c \sin \beta$$

$$\sin \beta = \sin \theta \times \frac{r}{l_c} = \frac{\sin \theta}{n}$$

## CHAPTER - 5

### BALANCING

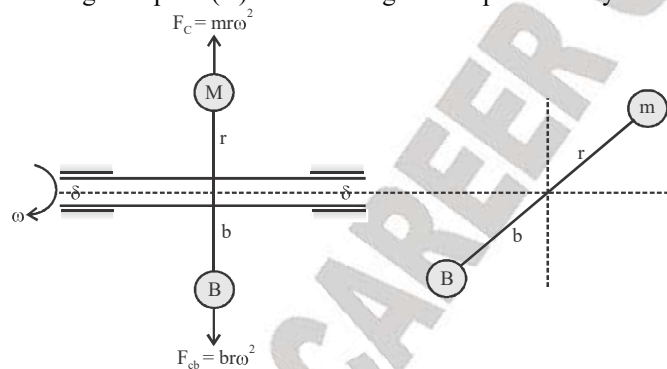
#### 5.1 INTRODUCTION

In an every machine there are two types of parts are commonly used as: rotating part and reciprocating part. It is necessary to balance both types of parts in the machines to avoid unbalance force components. These unbalanced focus case excessive noise, vibrations or and fear of the system.

Meanwhile now days high speed engines and other machines are requirement of each every industry. Thus, it is very essential to balance the machines to reduce or eliminate unbalanced forces.

#### 5.2 BALANCING OF A SINGLE ROTATING MASS

Whenever a certain mass is attached to a rotating shaft, it exists centrifugal force which tends to bend the shaft and produce vibration and also produce lends on bearings. To eliminate the effect of centrifugal force we should attach an another balanced mass in opposite directions in such a way that it eliminate effect of centrifugal force. Let us consider a mass ( $m$ ) is attached to the shaft which is rotating at an angular speed ( $\omega$ ) and centrifugal force produced by this mass is  $F_c$ . Thus



$$F_c = F_{cb}$$

$$mr\omega^2 = Bb\omega^2$$

$$mr = Bb$$



1. Product of mass  $\times$  radius will be remains same for both original mass and balanced mass respectively.
2. Usually radius of balanced mass is taken larger to reduce the balance mass.
3. Both mass will rotate at same angular speed.

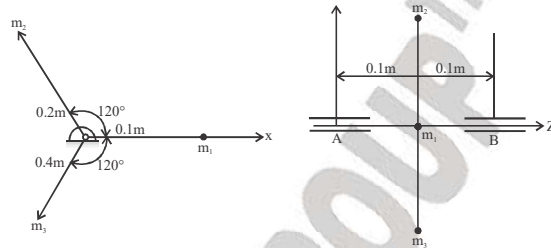
#### 5.3 EXTERNAL BALANCING OF SINGLE ROTATING MASS

If in any case balancing mass cannot possible to place just equal and opposite to original mass then external balancing of mass can be used. In such case, balanced mass to be placed in different plane but the balancing cannot be achieved by single mass because due to difference in planes the



**GATE QUESTIONS**

1. Three masses are connected to a rotating shaft supported on bearings A and B as shown in the figure. The system is in space where the gravitational effect is absent. Neglect the mass of shaft and rods connecting the masses. For  $m_1 = 10$  kg,  $m_2 = 5$  kg and  $m_3 = 2.5$  kg and for a shaft angular speed of 1000 radian/s, the magnitude of the bearing reaction (in N) at location B is \_\_\_\_\_



[GATE - 2017]

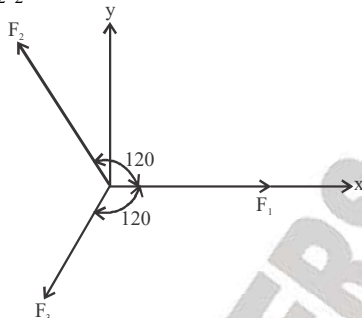
**SOLUTIONS**

**Sol. 1. (0)**

$$F_1 = m_1 r_1 \omega^2 = 10 \times 0.1 \times \omega^2 = \omega^2$$

$$F_2 = m_2 r_2 \omega^2 = 5 \times 0.2 \times \omega^2 = \omega^2$$

$$F_3 = m_3 r_3 \omega^2 = 2.5 \times 0.4 \times \omega^2 = \omega^2$$



$$\Sigma F_x = \omega^2 [1 + \cos 120^\circ + \cos 120^\circ] = 0$$

$$\Sigma F_y = F_2 \cos 30^\circ - F_3 \cos 30^\circ = 0$$

$$\therefore \text{Net force} = 0$$

$$\therefore \text{Bearing reactions} = 0 \text{ N}$$

## CHAPTER - 6

### GOVERNORS

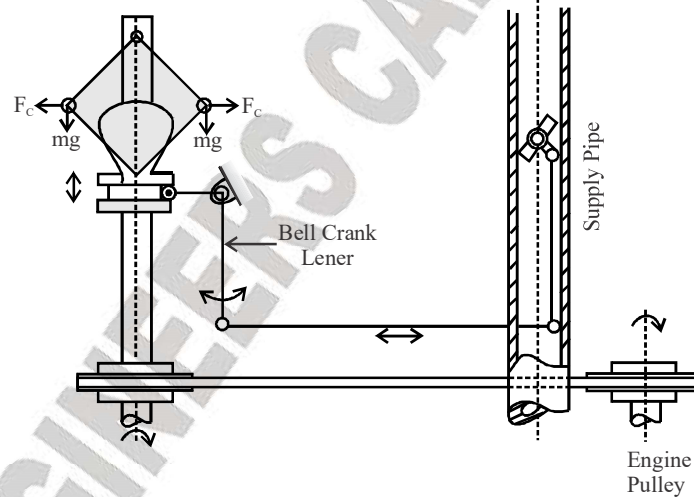
#### 6.1 INTRODUCTION

The flywheel which minimizes fluctuations of speed within the cycle but it cannot minimize fluctuations due to load variation. This means flywheel does not exercise any control over mean speed of the engine. To minimize fluctuations in the mean speed which may occur due to load variation, governor is used. The governor has no influence over cyclic speed fluctuations but it controls the mean speed over a long period during which load on the engine may vary.

When there is change in load, variation in speed also takes place then governor operates a regulatory control and adjusts the fuel supply to maintain the mean speed nearly constant. Therefore, the governor automatically regulates through linkages, the energy supply to the engine as demanded by variation of load so that the engine speed is maintained nearly constant.

The sketch of a governor along with linkages which regulates the supply to the engine. The governor shaft is rotated by the engine shown in the figure. If load on the engine increases the engine speed tends to reduce, as a result of which governor balls move inwards. This causes sleeve to move downwards and this movement is transmitted to the valve through linkages to increase the opening and, thereby, to increase the supply.

On the other hand, reduction in the load increases engine speed. As a result of which the governor balls try to fly outwards. This causes an upward movement of the sleeve and it reduces the supply. Thus, the energy input (fuel supply in IC engines, steam in steam turbines, water in hydraulic turbines) is adjusted to the new load on the engine. Thus the governor senses the change in speed and then regulates the supply. Due to this type of action it is simple example of a mechanical feedback control system which senses the output and regulates input accordingly.



Governor and Linkages

#### 6.2 CLASSIFICATION OF GOVERNORS

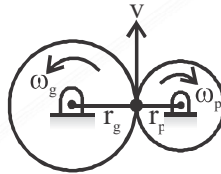
The broad classification of governor can be made depending on their operation.

1. Centrifugal governors
2. Inertia and flywheel governors
3. Pickering governors

**CHAPTER - 7****GEARS & GEAR TRAINS****7.1 INTRODUCTION**

Gears are used to transmit rolling and a sliding motion along the tangent at the point of contact. It can be successfully possible by the engagement of teeth.

The concept of gears has been derived from the rolling of two cylinders or disc. If there is no slip to be assumed in such case, that can definitely transmit motion of one to another and vice-versa. The rotating discs are known as friction wheels.



At a point of contact, the same linear velocity can be obtained as

$$v = \omega_g \cdot r_g = \omega_p \cdot r_p$$

$$v = 2\pi N_g r_g = 2\pi N_p r_p$$

$$\text{or } \frac{N_g}{N_p} = \frac{r_p}{r_g}$$

$$\text{or } \frac{\omega_g}{\omega_p} = \frac{r_p}{r_g}$$

It gives that, the speed of the two rolling discs without slipping is always proportional to the radii of the discs.

The friction wheels can be used to transmit motion at lower speeds. At high speeds it is not possible to transmit continuous motion without slipping. Thus, the concept of gear has been introduced to transmit motion and power at smaller centre distance. This lead to the formation of teeth on the discs and the motion between the surface changes from rolling to sliding. The disc with teeth are called as gear or gear wheel.

To obtain large reduction in velocity sometimes, two or more pair of gears may be used which is called as gear trains.

**7.2 ADVANTAGES OF GEARS**

Gear has many advantage over belt & chain drive.

1. It can transmit exact velocity ratio with positive drive.
2. It is capable of transmitting higher power.
3. It require less space, which gives compact layout.
4. It gives reliable service with high efficiency.

**7.3 CLASSIFICATION OF GEARS**

Gear can be classified according to their relative position of axes of the shaft as:

## CHAPTER - 8

### VIBRATIONS

#### 8.1 INTRODUCTION

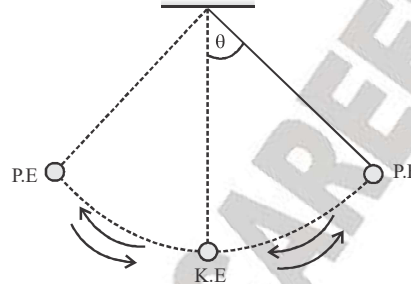
All bodies having mass and elasticity are capable of vibration. The mass is inherent of the body and elasticity can relative motion among its parts.

When body particles are displaced by the application of external force, the internal force in the form of elastic energy are present in the body. These forces try to bring the body to its original position.

At equilibrium position, the whole of the elastic energy is converted into kinetic energy and body continues to move in the opposite direction because of it. The whole of the K.E. is again converted into elastic or strain energy due to which the body again returns to the equilibrium position. In this way vibratory motion is repeated indefinitely and exchange of energy takes place.

This, any motion which repeats itself after an interval of time is called vibration or oscillation

**Example.** Simple pendulum, Spring mass system



The main reasons of vibration are as follows

1. Unbalanced centrifugal force in the system. This is caused because of non uniform material distribution in a rotating machine element.
2. Elastic nature of the system
3. External excitation applied on the system
4. Winds may cause vibrations of certain systems such as electricity lines, telephone lines etc.

#### 8.1.1 Advantages of Vibrations

1. Vibration can be used for useful purposes such as vibration testing equipments, vibrations conveyors, hoppers, sieves and compactors.
2. Vibration is found very fruitful in mechanical workshops such as in improving the efficiency of machining, casting, forging and welding techniques, musical instruments and earth quakes for geological research. Etc.
3. It is very useful for propagation of sound.

#### 8.1.2 Disadvantages of Vibration

1. The vibration causes rapid wear of m/c parts such as bearing and gears.
2. Unwanted vibrations may cause loosening of parts from the machine.
3. Many buildings, structures and bridges fall because of vibration.
4. Mechanical failure of the system if the frequency of excitation coincides with one of the natural frequency of the system, a condition of resonance is reached.
5. Sometimes because of heavy vibrations readings of instruments cannot be taken.

## ESE OBJ QUESTIONS

1. A simple spring- mass vibrating system has a natural frequency of  $N$ . If the spring stiffness is halved and the mass doubled, then the natural frequency will be  
 [ESE - 2017]  
 (a)  $0.5 N$  (b)  $N$   
 (c)  $2 N$  (d)  $4 N$
2. A car of mass  $1450 \text{ kg}$  is constructed on a chassis supported by four springs. Each spring has a force constant of  $40000 \text{ N m}$ . The combined mass of the two people occupying the car is  $150 \text{ kg}$ . What is the period of execution of two complete vibrations?  
 [ESE - 2017]  
 (a)  $0.63 \text{ s}$  (b)  $1.59 \text{ s}$   
 (c)  $4.96 \text{ s}$  (d)  $1.26 \text{ s}$
3. Consider the following statements:  
 Artefacts to prevent harmful effects resulting from vibrations of an unbalanced machine fixed on its foundation include  
 (i) Mounting the machine on springs thereby minimizing the transmission of forces.  
 (ii) Using vibration isolating materials to prevent or reduce the transmission of forces.  
 (iii) Moving the foundation so as to have only one degree of freedom towards reducing the transmission of forces.  
 Which of the above statements are correct?  
 [ESE -2017]  
 (a) (i) and (ii) only (b) (i) and (iii) only  
 (c) (ii) and (iii) only (d) (i), (ii) and (iii)
4. Two heavy rotors are mounted on a single shaft. Considering each of the rotors separately, the transverse natural frequencies are  $100 \text{ cycles/s}$  and  $200 \text{ cycles/s}$ , respectively. The lower critical speed will be  
 [ESE - 2017]  
 (a)  $12000 \text{ r.p.m.}$  (b)  $9360 \text{ r.p.m.}$   
 (c)  $8465 \text{ r.p.m.}$  (d)  $5367 \text{ r.p.m.}$
5. The equation of motion of a linear vibratory system with a single degree of freedom is  
 $4\ddot{x} + 9\dot{x} + 16x = 0$   
 The critical damping coefficient for the system is  
 [ESE - 2016]  
 (a)  $32$  (b)  $16$   
 (c)  $8$  (d)  $4$
6. A coil-spring of stiffness  $k$  is cut exactly at the middle and the two springs thus made are arranged in parallel to take up together a compressive load. The equivalent stiffness of the two springs is  
 [ESE - 2016]  
 (a)  $0.25 k$  (b)  $0.5 k$   
 (c)  $2 k$  (d)  $4 k$
7. A helical spring of  $10 \text{ N/mm}$  rating is mounted on top of another helical spring of  $8 \text{ N/mm}$  rating. The force required for a total combined deflection of  $45 \text{ mm}$  through the two springs is  
 [ESE - 2016]  
 (a)  $100 \text{ N}$  (b)  $150 \text{ N}$   
 (c)  $200 \text{ N}$  (d)  $250 \text{ N}$
8. The speed rating for turbine rotors is invariably more than  $\sqrt{2}$  times its natural frequency to  
 [ESE - 2015]  
 (a) Increase stability under heavy load and huge speed  
 (b) Isolate vibration of the system from the surrounding  
 (c) Minimize deflection under dynamic loading as well as to reduce transmissibility of force to the surrounding  
 (d) None of the above.
9. A block of mass  $10 \text{ kg}$  is placed at the free end of a cantilever beam of length  $1 \text{ m}$  and second moment of area  $300 \text{ mm}^4$ . Taking Young's modulus of the beam material as  $200 \text{ GPa}$ , the natural frequency of the system is  
 [ESE - 2015]  
 (a)  $30\sqrt{2} \text{ rad/s}$  (b)  $2\sqrt{3} \text{ rad/s}$   
 (c)  $3\sqrt{2} \text{ rad/s}$  (d)  $20\sqrt{3} \text{ rad/s}$

**GATE QUESTIONS**

1. The damping ratio for a viscously damped spring mass system, governed by the relationship

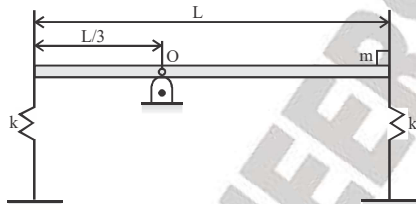
$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t), \text{ is given by}$$

[GATE - 2017]

- (a)  $\sqrt{\frac{c}{mk}}$
- (b)  $\frac{c}{2\sqrt{km}}$
- (c)  $\frac{c}{\sqrt{km}}$
- (d)  $\sqrt{\frac{c}{2mk}}$

2. A thin uniform rigid bar of length L and mass M is hinged at point O, located at a distance of  $\frac{L}{3}$  from one of its ends. The bar is

further supported using springs, each of stiffness k, located at the two ends. A particle of mass  $m = \frac{M}{4}$  is fixed at one end of the bar, as shown in the figure. For small rotations of the bar about O, the natural frequency of the system is

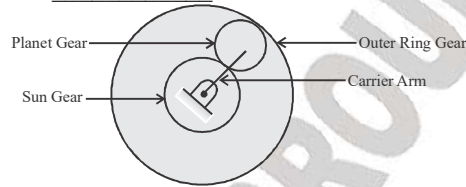


[GATE - 2017]

- (a)  $\sqrt{\frac{5k}{M}}$
- (b)  $\sqrt{\frac{5k}{2M}}$
- (c)  $\sqrt{\frac{3k}{2M}}$
- (d)  $\sqrt{\frac{3k}{M}}$

3. In an epicyclic gear train, shown in the figure, the outer ring gear is fixed, while the sun gear rotates counterclockwise at 100 rpm. Let the number of teeth on the sun, planet and outer gears to be 50, 25 and 100 respectively. The ratio of magnitudes of angular velocity of the

planet gear to the angular velocity of the carrier arm is \_\_\_\_\_.



[GATE - 2017]

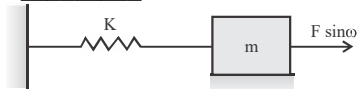
4. The radius of gyration of a compound pendulum about the point of suspension is 100 mm. The distance between the point of suspension and the centre of mass is 250 mm. Considering the acceleration due to gravity as  $9.81 \text{ m/s}^2$ , the natural frequency (in radian/s) of the compound pendulum is \_\_\_\_\_

[GATE - 2017]

5. The static deflection of a spring under gravity, when a mass of 1kg is suspended from it, is 1 mm. Assume the acceleration due to gravity  $g = 10 \text{ m/s}^2$ . The natural frequency of this spring-mass system (in rad/s) is \_\_\_\_\_

[GATE - 2016]

6. A single degree of freedom spring-mass system is subjected to a harmonic force of constant amplitude. For an excitation frequency of  $\sqrt{\frac{3k}{m}}$ , the ratio of the amplitude of steady state response to the static deflection of the spring is \_\_\_\_\_



[GATE - 2016]

7. A single degree of freedom mass-spring – viscous damper system with mass m, spring constant k and viscous damping coefficient q is critically damped. The correct relation among m, k and q is

[GATE - 2016]

- (a)  $q = \sqrt{2km}$
- (b)  $q = 2\sqrt{km}$